

Further investigation of the coherence of traffic noise transmission through an open window into a rectangular room of a high-rise building by considering time delay

Zhang Jiping¹, Sun Jiarong², Zhang Xin³, Wang Zheming⁴, Xu Mingzhu⁵
 Zhejiang Research and Design Institute of Environmental Protection,
 109 Tian Mu Shan Road, Hangzhou 310007 Zhejiang, China

ABSTRACT

Using wave theory, modal analysis, and a traffic noise model in the past, a theoretical method was proposed and experimentally verified for calculating the coherence between sound pressure inside and outside an open window of a rectangular room in a high-rise building. This paper presents further investigation of the coherence by considering the time delay between the sound pressure waves outside the window and those inside the research room. Firstly, the principle of the method is designed, derived, verified, and quantified by an outdoor experiment at a road whose traffic noise propagates at a distance, what is the simplest scenery of road traffic noise emission. Then, the theoretical principle is applied to the room. A defined experiment to verify and quantified the case will be tested in the future.

Keywords: Coherence of traffic noise, Time delay, Open window of a room for natural ventilation

I-INCE Classification of Subject Number: 50

LIST OF SYMBOLS

A_2	area of sound absorption except S	p_0	reference sound pressure
a_0	reference acceleration	r_i	distance between the point and the receiving sensor ($x_{2-0}, y_{2-0}, z_{2-0}$)
c	sound speed in air	R	room constant
c_g	sound speed in glass	S	area of the window
C_i	a correction term for the A-weighting of the sound reduction index in octave band i to the reference spectrum	S'	area of the room except S ,
E_1	energy density outside of the window	S_g	area of glass cross section in x and y axis directions, equals to the multiplication by variables thickness and wideness,
E_2	energy density inside of the window	S_i	area of one glass pane in the window
E_{gx}	energy density of x axis directions in glass	SD	standard deviation of signal time series

¹ jpzhang_daniel92@163.com,

² 22951667@qq.com,

³ zxin@zjhac.com,

⁴ 747533658@qq.com,

⁵ 68077483@qq.com.

E_{gy}	energy density of y axis directions in glass	TL	sound insulation level of an element (window, door, or wall)
f	signal frequency	TL_A	sound reduction index in dBA
I_g	sound intensity in glass	TL_i	sound reduction index in the i^{th} octave band:
I_i	sound intensity of one glass pane in the window	v	velocity of glass surface vibration in Z axis direction
K_{abs}	vibration transmission function in glass, here chooses and equals to $1/\rho_{\text{abs}}$	v_0	reference velocity
L_0	noise level in free sound field	v_{gx}	velocity of glass vibration in x axis direction
L_2	sound level in front of the window in room	v_{gy}	velocity of glass vibration in y axis direction
L_{2i}	sound level from any one point of the S	v'_{gxi}	velocity of glass vibration in x axis directions picked out by vibration sensor on glass surface
L_a	acceleration level	v'_{gyj}	velocity of glass vibration in y axis directions picked out by vibration sensor on glass surface
L_{ax}	acceleration levels inside glass in the x directions parallel to glass surface	W	sound power of the window in cross section S
L_{ay}	acceleration levels inside glass in the y axis directions parallel to glass surface	W_0	reference sound power
L_{az}	acceleration levels on the glass surface in the Z axis direction	W_g	sound power of the glass in cross section of x and y axis directions
L_{eq}	equivalent continuous sound pressure level	$(x_{2-0}, y_{2-0}, z_{2-0})$	receiving sensor position
L_N	$L_5, L_{95}, L_{\text{max}}, L_1, L_{10}, L_{50}, L_{90}, L_{99},$ and L_{min}	α	average absorption coefficient of the room
L_v	velocity level	α_i	absorption coefficient of the different surface in the room
L_w	sound power of the area S ,	τ	transmission coefficient
L_{wi}	sound power from any one point of the S	φ	direct contribution part of window area sound source S
M	total number of using vibration sensors, one unique triaxial vibration accelerometer is design and tested in this research	ρ_g	glass density
m	the m^{th} point umber of using vibration sensors, one unique triaxial vibration accelerometer is design and tested in this research	ρ_{abs}	energy efficiency of absorption in glass by vibration sensor in x and y axis directions
p	sound pressure on the frame of the window outside	ρ_{rad}	energy efficiency of radiation from glass surface towards the air space in room

1. INTRODUCTION

Coherence is a very important index to evaluate the acoustical characteristics by more than one signal channel, especially for active sound and vibration control. Many researches in this area theoretically consider and derive the coherence firstly, but the time delays between the actual channels in a real system may not be avoided in order to maintain and raise the active control efficiency.

An outdoor traffic noise propagating at a distance is the simplest scenery in environmental noise modelling and control. This paper will theoretically calculate and experiment the outdoor sound pressure at a distance d from a motor road and that at distance $d+D$ from the road and the coherence between the two points. Here we will not only prove or observe that the existing time delay may not be avoided and do worsen the coherence, but also estimate the quantity with values. At last, the principle of the method is applied to a wide application system that the road traffic noise enters into an open window of a rectangular room in a high-rise building.

In reference [1], using wave theory, modal analysis, and a traffic noise model, a theoretical method was early proposed and experimentally verified for calculating the coherence between sound pressure inside and outside an open window of a rectangular room in a high-rise building by the author of this paper. Then, the theoretical principle is applied to the room presenting further investigation of the coherence by considering the time delay between the sound pressure waves outside the window and those inside the research room. A defined experiment to verify and quantify the case will be tested in the future.

2. THEORY OF CONSIDERING TIME DELAY FROM OUTDOOR TRAFFIC SOUND PRESSURE AT A ROAD WHOSE TRAFFIC NOISE PROPAGATES AT A DISTANCE TO ENTERING A WINDOW OF A ROOM

2.1 Theory of outdoor traffic sound pressure at a road whose traffic noise propagates at a distance

(1) Outdoor traffic sound pressure at a road whose traffic noise propagates at a distance

A measuring point is at a traffic road as shown in Figure 1. The following assumptions are adopted: There are simple obstacles such as barrier between the point and the road; the road has two equivalent vehicle lanes, the traffic flow through each lane has identical averaged speed v , for three types of vehicles, including automobiles, heavy vehicles (including buses and trucks), and motorcycles, with the vehicles flow for the road being N per second, and the flow of each lane being $N_s=N/2$; the flow of the u^{th} type of vehicles is designated N_{su} , such that $N_s=\sum_{u=1}^3 N_{su}=N/2$. The vehicle flow within a particular segment $[-L_s, L_s]$ of the s^{th} lane, directly facing the measuring point, is n_s , and the flow of the u^{th} type of vehicle is n_{su} ($n_{su}=\sum_{u=1}^3 N_{su}$).

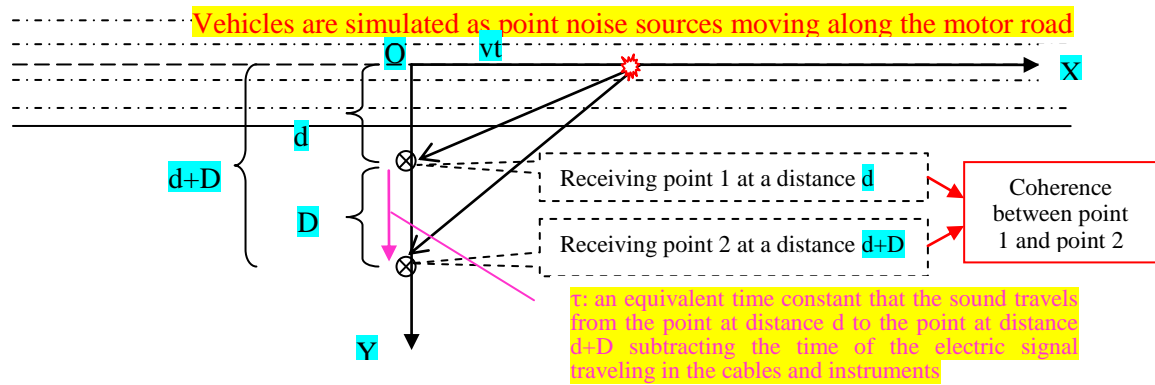


Figure 1: Layout diagram of the two noise-monitoring points at a road whose traffic noise propagates at a distance d and at a distance $d+D$

A vehicle can be represented by a monopole sound source moving along the road close to the earth ground surface when the observation point is far away from it. Therefore the sound pressure due to a vehicle at a reception position can be expressed in the frequency domain as follows: $p(r,\omega,t)=(1+C_r) \times A(\omega) \times m(\omega,r) \times \exp[i(\omega t-kr)]/4\pi r$, where $A(\omega)$ is the source strength related to the surface vibrating velocity of the vehicle and the characteristics of the air; C_r is the reflection coefficient of ground; r is the distance between the vehicle and the reception point; ω is the sound frequency ($=2\pi f$); t is time; k is the wave number ($=\omega/c$); $m(\omega,r)$ is the reduction due to air absorption or the influence of wind, temperature, and obstacles, which can be expressed as $m(\omega,r)=\exp(-M\omega) \times 10^{-0.1\Delta/2}$ in the simplest case (here, M denotes the air absorption coefficient, and Δ the barrier effectiveness reduction). The vehicles moving on the road are described by some statistical distribution (e.g., a Poisson distribution) so the sound pressure from the road at receiving point can be calculated in the followings.

$$p(\omega, t) = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n A_{suj} \frac{\exp[i(\omega t - k_{suj} r_{suj})]}{4\pi r_{suj}} \times m(\omega, r_{suj}), \quad (1)$$

$$\approx \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n A_{suj} \frac{\exp[i(\omega t - k_{suj} r_{suj})]}{4\pi r_{suj}} \times m(\omega, r_{suj});$$

$$E(A_{suj} \times A_{s'uj'}^*) = E(|A_{fsuj}|^2) \delta(s-s') \delta(u-u') \delta(j-j'). \quad (2)$$

Where, $r_{suj1} = \text{Sqrt}[d^2 + (vt)^2]$ and $r_{suj2} = \text{Sqrt}[(d+D)^2 + (vt)^2]$, the sound pressure $p(\omega, r, v, t)$ is assumed to be affected only by the vehicles in the time interval $[t-2Ls/v, t+Ls/v]$; j denotes an independent random time or vehicle location. Each complex independent random variable $A_{suj}(1+Cr) \times A \times m(\omega, r)$ corresponds to the temporal change of the u^{th} -type vehicles on the s^{th} lane with an arbitrary density function at the distance of r_{suj} . A_{suj}^* is the complex conjugate of A_{suj} , and the t_{uj} are independent random time variables for when the vehicle and its sound pressure appears. d is the distance between the road and the receiving point, v the average vehicle moving speed.

(2) Theoretically calculating the outdoor sound pressure at a distance d , that at $d+D$, and the coherence between them

When only in an ideal theoretical condition or no practical concept, the coherence $k(\omega)$ between the two receiving points at d and at $d+D$ can be

$$k^2[p_1(\omega, t) \times p_2^*(\omega, t)] = \frac{\left| \int_{-\infty}^{+\infty} p_1(\omega, t) \times p_2^*(\omega, t) dt \right|^2}{\int_{-\infty}^{+\infty} p_1(\omega, t) \times p_1^*(\omega, t) dt \times \int_{-\infty}^{+\infty} p_2(\omega, t) \times p_2^*(\omega, t) dt} =$$

$$\frac{\left| \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj1}) \times m(\omega, r_{suj2})}{16\pi^2} \int_{-\infty}^{+\infty} \frac{\exp[ik_{suj}(r_{suj2} - r_{suj1})]}{r_{suj1} \times r_{suj2}} dt \right|^2}{\sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj1})^2}{16\pi^2} \int_{-\infty}^{+\infty} \frac{1}{r_{suj1}^2} dt \times \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj2})^2}{16\pi^2} \int_{-\infty}^{+\infty} \frac{1}{r_{suj2}^2} dt}$$

$$\approx \frac{\left| \int_{-\infty}^{+\infty} \frac{\exp[ik_{suj}(r_{suj2} - r_{suj1})]}{r_{suj1} \times r_{suj2}} dt \right|^2}{\int_{-\infty}^{+\infty} \frac{1}{r_{suj1}^2} dt \times \int_{-\infty}^{+\infty} \frac{1}{r_{suj2}^2} dt} \text{ (note: } m(\omega, r_{suj}) \text{ is simplified as the same so removed)} =$$

$$\frac{d(d+D)v^2}{\pi^2} \left| \int_{-\infty}^{+\infty} \frac{\exp[ik_{suj}(\sqrt{(d+D)^2 + (vt)^2} - \sqrt{d^2 + (vt)^2})]}{\sqrt{(d+D)^2 + (vt)^2} \times \sqrt{d^2 + (vt)^2}} dt \right|^2.$$

When acture measurement is made lasting time T and no time delay is considered temporarily, the above coherence calculation can have an evolution as

$$k^2[p_1(\omega, t) \times p_2^*(\omega, t)] = \frac{\left| \int_0^T p_1(\omega, t) \times p_2^*(\omega, t) dt \right|^2}{\int_0^T p_1(\omega, t) \times p_1^*(\omega, t) dt \times \int_0^T p_2(\omega, t) \times p_2^*(\omega, t) dt}$$

$$= \frac{\left| \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj1}) \times m(\omega, r_{suj2})}{16\pi^2} \int_0^T \frac{\exp[ik_{suj}(r_{suj2} - r_{suj1})]}{r_{suj1} \times r_{suj2}} dt \right|^2}{\sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj1})^2}{16\pi^2} \int_0^T \frac{1}{r_{suj1}^2} dt \times \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj2})^2}{16\pi^2} \int_0^T \frac{1}{r_{suj2}^2} dt}$$

$$\approx \frac{\left| \int_0^T \frac{\exp[ik_{suj}(r_{suj2} - r_{suj1})]}{r_{suj1} \times r_{suj2}} dt \right|^2}{\int_0^T \frac{1}{r_{suj1}^2} dt \times \int_0^T \frac{1}{r_{suj2}^2} dt} \text{ (note: } m(\omega, r_{suj}) \text{ is simplified as the same so it can be removed)}$$

$$= \frac{d(d+D)v^2}{\arctg\left(\frac{vT}{d}\right) \times \arctg\left(\frac{vT}{d+D}\right)} \times \left| \int_0^T \frac{\exp[ik_{suj}(\sqrt{(d+D)^2 + (vt)^2} - \sqrt{d^2 + (vt)^2})]}{\sqrt{(d+D)^2 + (vt)^2} \times \sqrt{d^2 + (vt)^2}} dt \right|^2,$$

where, $k(\omega)$ is the coherence between the two receiving points. The integral in Equation (4) should be calculated by numerical calculation.

In addition, $L_p(\omega, t) = 10\lg[p(t) \times p^*(t)/p_0^2]$,

$$L_{peq}(\omega, T) = 10\lg\left[\frac{1}{T} \int_0^T p(t) \times p^*(t) dt / p_0^2\right] = 10\lg\left[\frac{1}{T} \int_0^T \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj})^2}{16\pi^2 p_0^2 r_{suj}^2} dt\right], \quad (5)$$

$$L_{eq1}(d) = 10\lg\left[\frac{1}{T} \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{fsuj}|^2 \times m(\omega, r_{suj})^2}{16\pi^2 p_0^2} \times \frac{\arctg\left(\frac{vT}{d}\right)}{vd}\right] \quad (6)$$

$$L_{eq2}(d+D) = 10\lg \left[\frac{1}{T} \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{fsuj}|^2 \times m(\omega, r_{suj})^2}{16\pi^2 p_0^2} \times \frac{\arctg(\frac{vT}{d+D})}{v(d+D)} \right]. \quad (7)$$

Where, the parameters T, d, D, $L_{eq1}(d)$ and $L_{eq2}(d+D)$ can be easily measured, the v can be determined by Equations (6) and (7).

(3) The coherence considering time delay

When acture measurement is made while an equivalent time delay τ is also considered, we have

$$\begin{aligned} k^2[p_1(\omega, t) \times p_2^*(\omega, t + \tau)] &= \frac{\left| \int_0^T p_1(\omega, t) \times p_2^*(\omega, t + \tau) dt \right|^2}{\int_0^T p_1(\omega, t) \times p_1^*(\omega, t) dt \times \int_0^T p_2(\omega, t + \tau) \times p_2^*(\omega, t + \tau) dt} \\ &= \frac{\left| \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj1}) \times m(\omega, r_{suj2})}{16\pi^2} \int_0^T \frac{\exp[ik_{suj}(r_{suj2} - r_{suj1}) - i\omega\tau]}{r_{suj1}(t) \times r_{suj2}(t + \tau)} dt \right|^2}{\sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj1})^2}{16\pi^2} \int_0^T \frac{1}{r_{suj1}^2(t)} dt \times \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj2})^2}{16\pi^2} \int_0^T \frac{1}{r_{suj2}^2(t + \tau)} dt} \\ &\approx \frac{\left| \int_0^T \frac{\exp[ik_{suj}(r_{suj2} - r_{suj1})]}{r_{suj1} \times r_{suj2}} dt \right|^2}{\int_0^T \frac{1}{r_{suj1}^2(t)} dt \times \int_0^T \frac{1}{r_{suj2}^2(t + \tau)} dt} \quad (\text{note: } m(\omega, r_{suj}) \text{ is simplified as the same so removed}) \\ &= \frac{d(d+D)v^2}{\arctg(\frac{vT}{d}) \times \left[\arctg(\frac{vT}{d+D}) + \arctg(\frac{vT}{d+D}) \right]} \times \left| \int_0^T \frac{\exp[ik_{suj}(\sqrt{(d+D)^2 + (vt+vt)^2} - \sqrt{d^2 + (vt)^2}) - i\omega\tau]}{\sqrt{(d+D)^2 + (vt+vt)^2} \times \sqrt{d^2 + (vt)^2}} dt \right|^2, \quad (8) \end{aligned}$$

where, τ is an equivalent time constant that sound travels from the point at distance d to the point at distance d+D subtracting the time of the electric signal traveling in the cables and instruments. The integral in Equation (8) should be calculated by numerical calculation. From Equation (8), we can see that the bigger value τ is, the smaller the k^2 will be. The equivalent time delay τ can and will interfere the coherence of the system.

Again, $L_p(\omega, t) = 10\lg[p(t) \times p^*(t)/p_0^2]$

$$L_{peq}(\omega, T) = 10\lg \left[\frac{1}{T} \int_0^T p(t + \tau) \times p^*(t + \tau) dt / p_0^2 \right] = 10\lg \left[\frac{1}{T} \int_0^T \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{suj}|^2 \times m(\omega, r_{suj})^2}{16\pi^2 p_0^2 r_{suj}^2(t + \tau)} dt \right], \quad (9)$$

$$L_{eq1}(d) = 10\lg \left[\frac{1}{T} \int_0^T p(t) \times p^*(t) dt \right] = 10\lg \left[\frac{1}{T} \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{fsuj}|^2 \times m(\omega, r_{suj})^2}{16\pi^2 p_0^2} \times \frac{\arctg(\frac{vT}{d})}{vd} \right], \quad (10)$$

$$L_{eq2}(d+D) = 10\lg \left[\frac{1}{T} \int_0^T p(t + \tau) \times p^*(t + \tau) dt \right] = 10\lg \left[\frac{1}{T} \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=1}^n \frac{|A_{fsuj}|^2 \times m(\omega, r_{suj})^2}{16\pi^2 p_0^2} \times \frac{\arctg(\frac{vT-v\tau}{d+D}) + \arctg(\frac{vT}{d+D})}{v(d+D)} \right]. \quad (11)$$

Comparing Equations (9) - (11) with Equations (5) - (7) when T is bigger, they show that the time delay is not a problem for energy accumulation and average. This indicates the synchronous is an very important factor for transient active research, but it is not important for a long time energy impact assessment, such as lasting for 10-20 min..

2.2. Theory of traffic noise entering a window of a room by considering time delay

(1) Theoretically calculating the sound pressure inside a rectangular room in a high-rise building and that outside the open window of the room

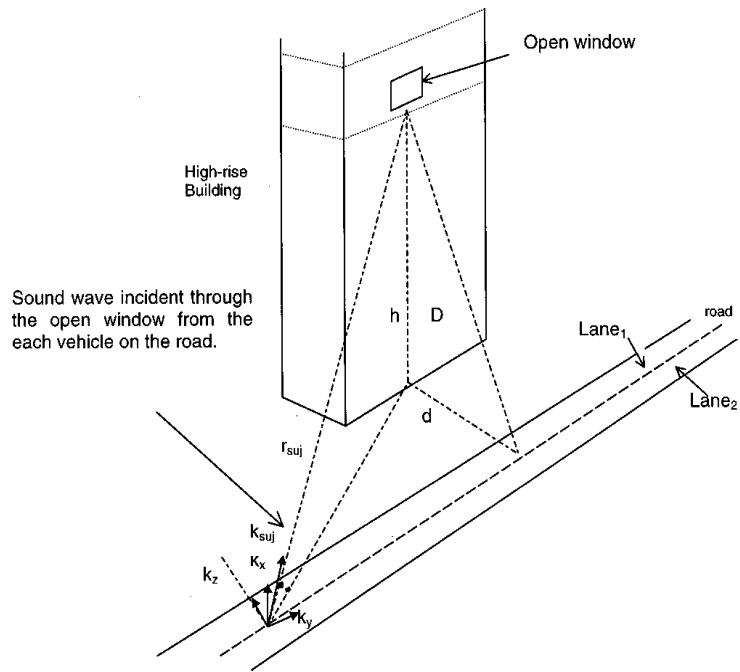
See Figure 2, using wave theory, modal analysis, and a traffic noise model, a method for theoretically calculating the sound pressure inside a rectangular room in a high-rise building and that outside the open window of the room was proposed and verified by an experiment in the author's paper^[1].

$$p(r, \omega) = (1 + Cr) \times A(\omega) \times m(\omega, r) \exp[i(\omega t - kr)] / 4\pi r, \quad (12)$$

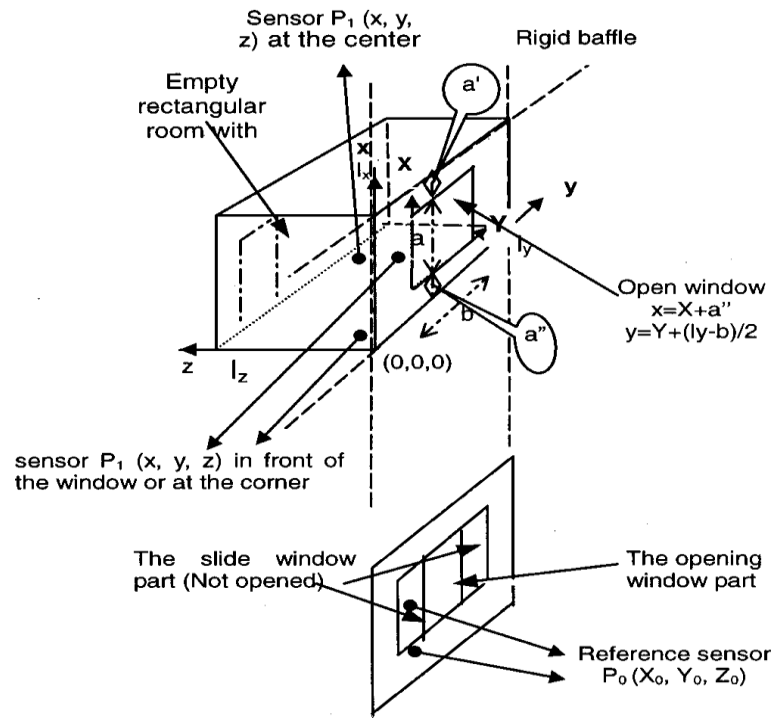
$$p(\omega, t) = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} A_{sju} \frac{\exp[i(\omega t - k_{suj} r_{suj})]}{4\pi r_{suj}} \approx \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{n_{su}} A_{sju} \frac{\exp[i(\omega t - k_{suj} r_{suj})]}{4\pi r_{suj}}, \quad (13)$$

$$E(A_{suj} \times A_{s'u'j}') = E(|A_{suj}|^2) \delta(s - s') \delta(u - u') \delta(j - j'), \quad (14)$$

$$\text{where, } A_{su0}=0 \text{ for all } s, u, \text{ and } r_{suj} = \sqrt{D^2 + v^2(t - t_{uj})^2}, \quad D = \sqrt{d^2 + h^2}. \quad (15)$$



Sketch of a high-rise building at a traffic road.



Sketch of the open window room with coordinates, parameters, and sensor locations.

* The symbols in Figures and Equations of traffic noise entering a window of a room in this paper are carefully illustrated in this Figure 2 and listed at the beginning of this paper.

Figure 2 - A theoretical coherence of traffic noise transmission through an open window into a rectangular room in high-rise buildings

$A_{suj} = (1 + Cr) \times A \times m(\omega, r)$, corresponds to the transient or temporal change of the u^{th} type vehicles on the s^{th} lane with an arbitrary density function at the distance of r_{suj} , and

$$P(x, y, z, \omega) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \exp(-ik_{mn}l_z) \cos\left(\frac{m\pi x}{l_x}\right) \cos\left(\frac{n\pi y}{l_y}\right) \cos[k_{mn}(l_z - z)] \exp(i\omega t), \quad (16)$$

$$k_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{l_x}\right)^2 - \left(\frac{n\pi}{l_y}\right)^2},$$

$$p_s \approx \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} \frac{2A_{suj}}{4\pi r_{suj}} \exp[i(\omega t - k_{suj}r_{suj})] \exp[-i(k_x X + k_y Y)] \cos(k_z Z). \quad (17)$$

Where, the symbols are listed at the beginning of this paper and shown in Figure 2.

(2) Theoretically calculating coherence between the sound pressure inside a rectangular room in a high-rise building and that outside the open window of the room

When only in the ideal theoretical condition or non practical concept, the coherence $k(\omega)$ between the two receiving points, from the prediction of the roadside ground noise of the vertical plane sound source in the far field to that inside an open widow room, can be

$$\int_{-\infty}^{+\infty} p_s \times p_s^* dt = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} \frac{4|A_{suj}|^2}{16\pi^2} \int_{-\infty}^{+\infty} \frac{\cos^2(k_z Z_0)}{r_{suj}^2} dt, \quad (18)$$

$$\int_{-\infty}^{+\infty} p(x, y, z, \omega) \times p^*(x, y, z, \omega) dt = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} \frac{4|A_{suj}|^2}{16\pi^2} \frac{1}{a^2 b^2} \left[H^T C^{-T} \left(\int_{-\infty}^{+\infty} D_1 \times (D_1^T)^* dt \right) (C^{-1})^* H^* \right], \quad (19)$$

$$\int_{-\infty}^{+\infty} p(x, y, z, \omega) \times p_s^* dt = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} \frac{4|A_{suj}|^2}{16\pi^2} \frac{1}{ab} \left[H^T C^{-T} D_2 \right], \quad (20)$$

$$k^2(\omega) = \left(\left| \int_{-\infty}^{+\infty} p(x, y, z, \omega) \times p_s^* dt \right|^2 \right) / \left(\int_{-\infty}^{+\infty} p_s \times p_s^* dt \times \int_{-\infty}^{+\infty} p(x, y, z, \omega) \times p^*(x, y, z, \omega) dt \right) \\ = \left(\left| H^T C^{-T} D_2 \right|^2 \right) / \left(\int_{-\infty}^{+\infty} \frac{\cos^2(k_z Z_0)}{r_{suj}^2} dt \times \left[H^T C^{-T} \left(\int_{-\infty}^{+\infty} D_1 \times (D_1^T)^* dt \right) (C^{-1})^* H^* \right] \right), \quad (21)$$

where, $r_{suj} = r(d, h, vt)$, defined in Equations (15); $[C] = [C(l_x, l_y, l_z; a, b, a'')]$, the matrix of coefficients defined in the Appendix; $[H] = [H(l_x, l_y, l_z)]$ and $[D1] = [D1(a, b)]$, both the vectors defined in the Appendix respectively. The integrals in Equation (21) should be calculated by numerical calculation.

(3) Traffic noise transmission through an open window into a rectangular room in high-rise buildings including time delay

When acture measurement is made while the equivalent time delay τ is also considered, the $k(\omega)$ become

$$\int_0^{\tau} p_s \times p_s^* dt = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} \frac{4|A_{suj}|^2}{16\pi^2} \int_0^{\tau} \frac{\cos^2(k_z Z_0)}{r_{suj}^2} dt, \quad (22)$$

$$\int_0^{\tau} p(x, y, z, \omega) \times p^*(x, y, z, \omega) dt = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} \frac{4|A_{suj}|^2}{16\pi^2} \frac{1}{a^2 b^2} \left[H^T C^{-T} \left(\int_0^{\tau} D_1 \times (D_1^T)^* dt \right) (C^{-1})^* H^* \right], \quad (23)$$

$$\int_0^{\tau} p(x, y, z, \omega, t + \tau) \times p_s^*(\omega, t) dt = \sum_{s=1}^2 \sum_{u=1}^3 \sum_{j=0}^{N_{su}} \frac{4|A_{suj}|^2}{16\pi^2} \frac{1}{ab} \left[H^T C^{-T} D_2 \right], \quad (24)$$

$$k^2(\omega) = \frac{\left| \int_0^{TT} p(x, y, z, \omega, t + \tau) \times p_s^*(\omega, t) dt \right|^2}{\int_0^{TT} p_s \times p_s^* dt \times \int_0^{TT} p(x, y, z, \omega) \times p^*(x, y, z, \omega) dt} = \frac{\left| \exp[-i\omega\tau] H^T C^{-T} D_2(t + \tau) \right|^2}{\int_0^{TT} \frac{\cos^2(k_z Z_0)}{d^2 + h^2 + (vt)^2} dt \times \left[H^T C^{-T} \left(\int_0^{TT} D_1(t + \tau) \times (D_1^T(t + \tau))^* dt \right) (C^{-1})^* H^* \right]} \quad (25)$$

Where, τ is an equivalent time constant that sound travels from the point on the window to the point at the room subtracting the time of the electric signal traveling in the cables and instruments. From Equation (25), we can see that the bigger value τ is, the smaller the k^2 will be. The time delay τ can and will interfere the coherence of the practical system. TT in Equation (23)-(25) is the time T defined similarly in Equation (4)-(11). Because the T in (23)-(25) indicates the matrix transpose, here uses TT expression instead of the time T. The integral in Equation (25) should be calculated by numerical calculation.

3. EXPERIMENTAL TEST AND ANALYSIS

3.1 Experiment

3.1.1 Experiment for decoupling of the outdoor sound pressure at a distance d and that at d distance d+D and the coherence between two points

(1) Experimental arrangement

In Figure 1, in order to show and verify the time delay, the environmental traffic noise fluctuation was observed by directly setting two representative points at a road whose traffic noise propagates at a distance, not only indirectly and remotely sensing the rise and fall of the road traffic noise, but also reflecting traffic and speed, and the two experimental points were measured synchronously. The site is at a busy motor road in Hangcity of China. The distance d from the first monitoring point to the center of the road is 40 m, and the distance D from the first monitoring point to the second monitoring point is 10 m.

(2) Measuring Methods

The measuring methods and procedures are referred to the related national criteria in China, including (a) GB3096-2008 *Environmental quality standard for noise*, (b) HJ640-2012 *Technical specifications for environmental noise monitoring-Routine monitoring for urban environmental noise*, (c) GB22337-2008 *Emission Standard for community noise*, (d) GB12348-2008 *Emission Standard for industrial enterprises noise at boundary*, (e) GB3096-1993 *Standard of environmental noise of urban area*, and (f) GB/T14623-1993 *Measuring method of environmental noise of urban area*.

(3) Measuring Instruments and Focused Metrics

One automatic environmental acoustics precise monitoring analysis system was applied. The system was a COCO80 dynamic signal analyzer made by Crystal Instruments, USA, primarily monitoring the sound signals of the traffic noise and background natural environmental noise. The sensors were two pairs of 1/2-inch PCB types of frequency ranging from 3.5 Hz to 20 kHz with amplifier PCB378B02. The coupled channels were operated synchronously, and their sound sensors were protected by windscreens. Before and after the monitoring, the devices were calibrated by a B&K standard sound source. When the device was in use, the automatic monitoring lasted for

20 min each time, including recording original signals in the time stream. The recorded data were brought back to the laboratory and used digital signal processing for any acoustical purpose. Actually, only few represented transient data are chosen for the requirement of coherence calculation.

(4) Measuring dates and durations

The experiment took place on March 7, 2019, and each test duration lasted for 10~20 min according to related standards. The tests were done in daytime. The weather was sunny and had little wind.

(5) Measuring Results and Analysis

The results of measured coherence and the processed coherence from the measured coherence are shown in Figure 3. The process method is decreasing some delay time in the channel closer to the motor road by inserting multi-order FIR digital filters. The effectiveness of this operating equals to a time data shifting to the all time serials in the first channel.

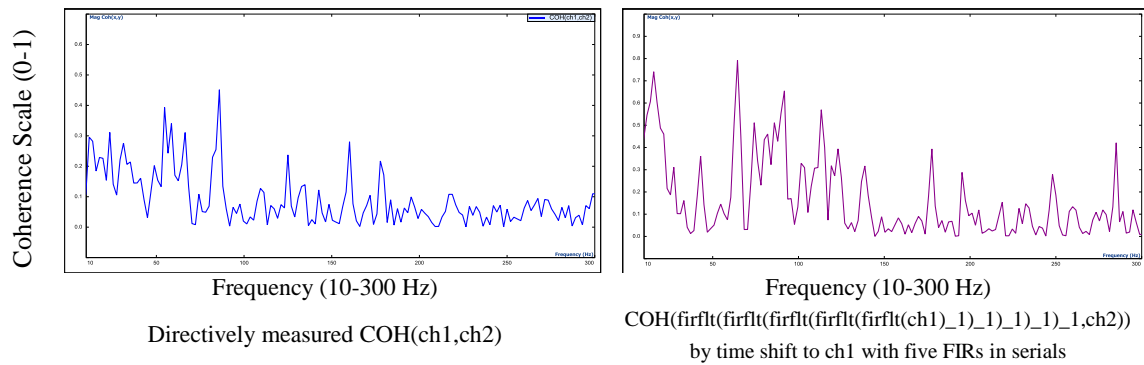


Figure 3 - Comparing the coherences measured and processed at the two points in Figure 1

Only the low frequency signals are attentioned in the experiment for the cases of road traffic noise propagation towards a far distance at motor road, or traffic noise transmission through an open window into a rectangular room of a high-rise building. The time delay of one FIR digital filter $\tau=N/(2f_s)$ ^[2], where N is the order of the FIR filter and f_s is the cut-off frequency of the lower pass filter. Generally, the lower pass FIR filter is chosen. The sampling frequency is resampling the data at the rate of 3-4 times the f_s which meets the Nyquist Theorem.

In Figure 3, we can see the influence from the time delay by the quantity of values. In the evaluation of an active control and transient sound signals application, it can be post processed easily. However, it is not easy for a real and live system, because of the time cost in the process. The advantage is that the related coefficients can be gotten on line.

3.1.2 Experiment for decoupling of traffic noise transmission through an open window into a rectangular room in high-rise buildings including time delay

See Figure 4. The schematic of an experiment for investigating the coherence of traffic noise transmission through an open window into a rectangular room in high-rise buildings, similar in reference 1, is planned for the future verification and simulations. The difference between the past and the future experiment is the time delay will be focused on. The defined experiment to verify and quantify the case will be tested in future because of a short of the time length to prepare for this paper.

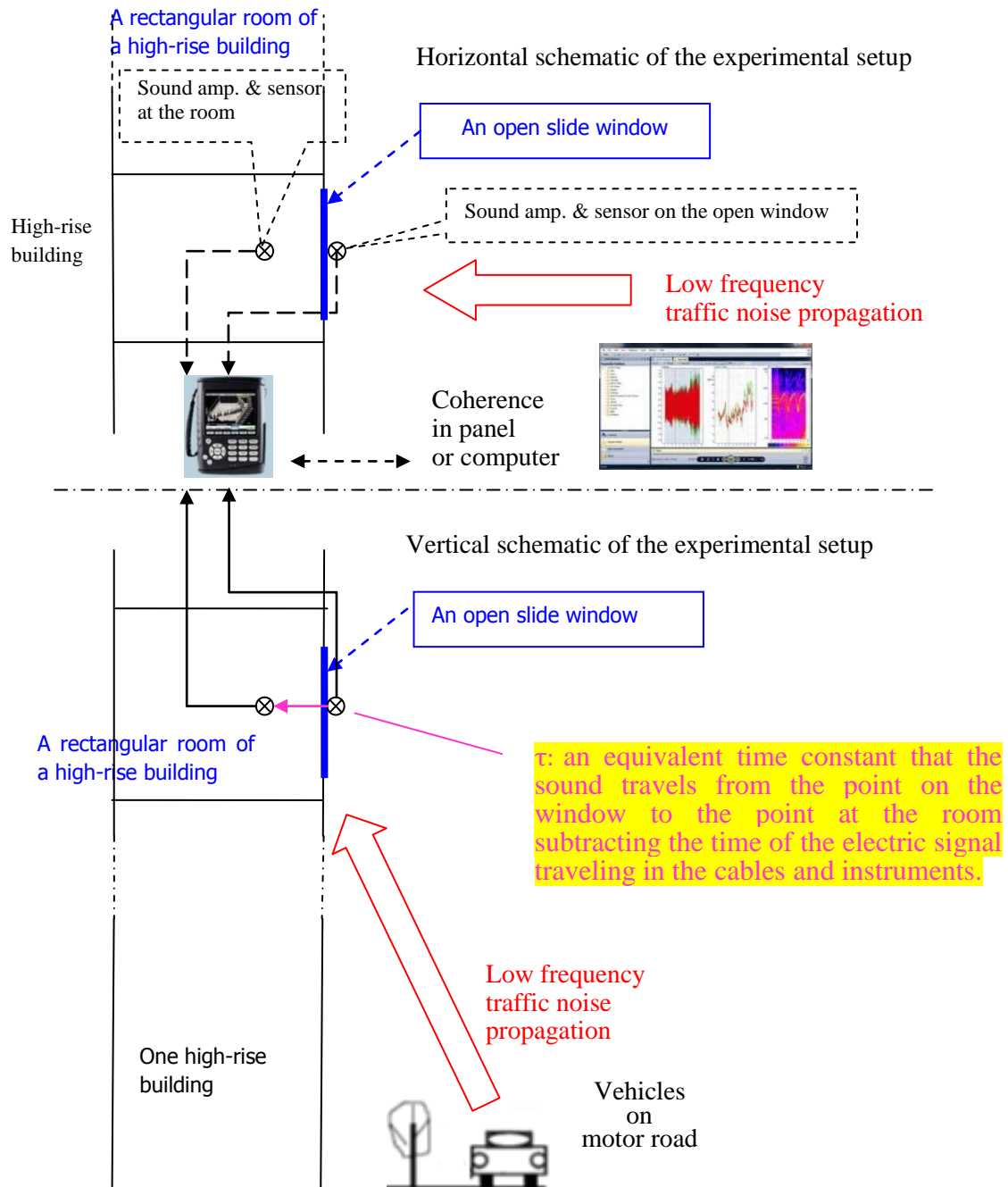


Figure 4 - Scheduled schematic of experimental setup with the apparatus and the sensors for investigating coherence of traffic noise transmission through an open window into a rectangular room of a high-rise building for future simulations and verifications including time delay

4. CONCLUSIONS AND DISCUSSION

In order to research the wide sceneries of road traffic noise entering into an open window of a rectangular room in a high-rise building by considering time delay, the principle was firstly observed and the quantity of values were estimated in an experiment of the outdoor sound pressure at a distance d from a motor road and that at distance $d+D$ from the road and the coherence between the two points. Then, it was applied to the room presenting further investigation of the coherence by considering the time delay between the sound pressure waves outside the window and those inside the research room. A defined experiment to verify and quantify the case will be tested in the future.

In the outdoor experiment, calculate Equation (8)/ Equation (4), let the time T be bigger enough, $v= 40$ km/h (in the real site experment, and have it 0, 20, 40, 60, 80 km/h respectively for an observation), $d=40$ m, $D=10$ m (in the real site experment, and have it changing from 0-100 m for an observation), $\tau \approx D/c$, and $c=340$ m/s, except of the two parts of integrals or assume both integrals be the same approximately, the rate of Equation (8) over Equation (4) will be in Equation (26) and in Figure 5.

$$\text{Rate of } \frac{\text{Equation(8)}}{\text{Equation (4)}} \text{ at least} = \frac{\frac{\pi}{2}}{\frac{\pi}{2} + \arctg\left(\frac{v\tau}{d+D}\right)} - \frac{\frac{\pi}{2}}{\frac{\pi}{2} + \arctg\left[\frac{vD}{c(d+D)}\right]} \quad (26)$$

Where, the result of Figure 5 shows: ① The equivalent constant τ rises when the distance D between the two measuring points, at least the rate (not considering the 2 integrals) will be a discount to the theoretical coherence $k(\omega)$, that is the $k(\omega)$ is worsen by τ . In order to maintain even increase $k(\omega)$, the strategy in the design of active control or monitoring should balance this important factor. ② With the τ , the speed v of vehicles' moving on the motor road is also an important related factor which may influence the $k(\omega)$, that is the higher the speed is, the smaller the Rate is. The Rate can decrease the $k(\omega)$ by τ and v together, although the related value of the Rate is small, which is between 0.965-1.000.

In the suggested future experiment in Figure 4 to verify traffic noise transmission through an open window into a rectangular room in high-rise buildings, the equivalent time delay τ and its quantity will be focused on again.

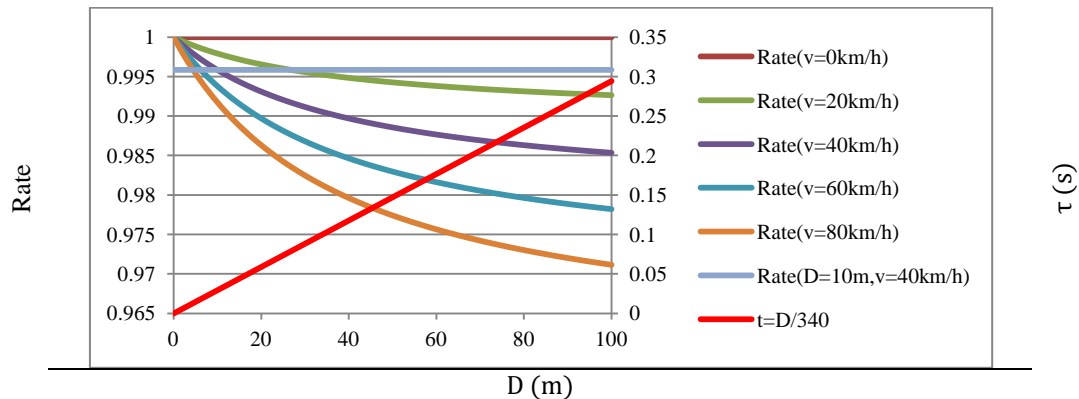


Figure 5 - An estimated Rate to the coherence $k(\omega)$ by a case motor road, where vehicles moving speed is 40km/h; the distance d , between the center line of the road and the measuring point 1, is 40 m; and the distance D , between the measuring point 1 and the measuring point 2, is changing from 0-100 m and $D=10$ m respectively

5. ACKNOWLEDGEMENTS

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2. Hongmei Zhang, Wangang Han, Time-delay analysis of FIR filter and its application to signal fusion, *Engineering Journal of Wuhan University*, Vol.(No.2), Apr. 2016, 303-308 (2016). (in Chinese)

7. REFERENCES APPENDIX: MATRIX AND VECTOR DEFINITIONS

a. $[C]=[C(\mathbf{l}_x, \mathbf{l}_y, \mathbf{l}_z; \mathbf{a}, \mathbf{b}, \mathbf{a}'')]$

$$[C] = [C_{00}(p, q), C_{01}(p, q), \dots, C_{0N}(p, q), C_{10}, \dots, C_{1N}, \dots, C_{mn}(p, q), \dots, C_{N0}, \dots, C_{NN}]^T$$

$$C_{mn}(p, q) = [C_1(p, q)\delta(p-m)\delta(q-n) + C_2(m, n)C_3(m, n)] \quad (A-1)$$

where, $C_1(p, q) = \exp(-ik_{pq}l_z) \cos(k_{pq}l_z) I_1(p) I_2(q)$

$$C_2(m, n) = KK' \frac{l_x l_y}{2\pi a^2 b^2} k_{mn} \exp(-ik_{mn}l_z) \sin(k_{mn}l_z) II_1(m) II_2(n)$$

$$I_1(m) = 1 \quad (m = 0) = \frac{l_x}{m\pi(a-l_x)} \sin\left(\frac{am\pi}{2l_x} - \frac{m\pi}{2}\right) \cos\left[\frac{m\pi(a+2a'')}{2l_x} - \frac{m\pi}{2}\right] +$$

$$\frac{l_x}{m\pi(a+l_x)} \sin\left(\frac{am\pi}{2l_x} + \frac{m\pi}{2}\right) \cos\left[\frac{m\pi(a+2a'')}{2l_x} + \frac{m\pi}{2}\right] \quad (m \neq 0)$$

$$I_2(n) = 1 \quad (n = 0) = \frac{l_y}{n\pi(b-l_y)} \sin\left(\frac{bn\pi}{2l_y} - \frac{n\pi}{2}\right) + \frac{l_y}{n\pi(b+l_y)} \sin\left(\frac{bn\pi}{2l_y} + \frac{n\pi}{2}\right) \cos[n\pi] \quad (n \neq 0)$$

$$II_1(m) = 1 \quad (m = 0) = \frac{a}{m\pi} \left\{ \frac{\sin\left(\frac{am\pi}{2l_x} - \frac{m\pi}{2}\right) \cos\left[\frac{1}{2}\left(\frac{a+2a''}{l_x} + \frac{a}{a}\right)m\pi - \frac{m\pi}{2}\right]}{a-l_x} + \frac{\sin\left(\frac{am\pi}{2l_x} + \frac{m\pi}{2}\right) \cos\left[\frac{1}{2}\left(\frac{a-2a''}{l_x} - \frac{a}{a}\right)m\pi + \frac{m\pi}{2}\right]}{a+l_x} \right\} (m \neq 0)$$

$$II_2(n) = 1 \quad (n = 0) = \frac{b}{n\pi} \left\{ \frac{\sin\left(\frac{bn\pi}{2l_y} - \frac{n\pi}{2}\right) \cos\left[\frac{1}{2}\left(\frac{b+l_y}{l_y} + \frac{b}{b}\right)n\pi - n\pi\right]}{b-l_y} + \frac{\sin\left(\frac{bn\pi}{2l_y} + \frac{n\pi}{2}\right) \cos\left[\frac{1}{2}\left(\frac{b-l_y}{l_y} - \frac{b}{b}\right)n\pi + n\pi\right]}{b+l_y} \right\} (n \neq 0)$$

$$C_3(m, n) = \int_S \int_{S'} \cos\left(\frac{m\pi}{a} X\right) \cos\left(\frac{n\pi}{b} Y\right) \cos\left(\frac{m\pi}{a} X'\right) \cos\left(\frac{n\pi}{b} Y'\right) \exp(-ikr_z) dS dS' / r_z$$

For the separated part in which ($r_z \rightarrow 0$) or ($X \rightarrow X'$ and $Y \rightarrow Y'$)

$$\approx \frac{a}{step_1} \frac{a}{step_2} \frac{b}{step_1} \frac{b}{step_2} \int_0^{step_1} \int_0^{step_2} \int_0^{step_1} \int_0^{step_2} \cos^2\left(\frac{m\pi X}{a}\right) \cos^2\left(\frac{n\pi Y}{b}\right) \frac{\exp(-ikr_z)}{r_z} dXdYdX'dY'$$

$$= \left(\frac{ab}{step_1 \times step_2}\right)^2 \int_0^{step_1} \int_0^{step_2} \cos^2\left(\frac{m\pi X}{a}\right) \cos^2\left(\frac{n\pi Y}{b}\right) dXdY \int_0^{2\pi} \int_0^R \exp(-ikr_z) d\theta dr_z$$

$$= \frac{2\pi}{k} \left(\frac{ab}{step_1 \times step_2}\right)^2 [\exp(-ikR) - 1] \left[\frac{step_1 + a}{2} + \frac{a}{4m\pi} \sin\left(\frac{2m\pi \times step_1}{a}\right)\right] \left[\frac{step_2 + b}{2} + \frac{b}{4n\pi} \sin\left(\frac{2n\pi \times step_2}{b}\right)\right]$$

where, $step_1$ and $step_2$ denote the small unit by dividing the parameters a and b respectively.

Because $dS = dS' \approx \pi R^2 \approx step_1 \times step_2$ $R = \sqrt{step_1 \times step_2 / \pi}$

b. $[D1]=[D1(\mathbf{a}, \mathbf{b})]$

$$\overset{\omega}{D}_1 = [D_1] = (D_{1(00)}, D_{1(01)}, D_{1(02)}, \dots, D_{1(0N)}, D_{1(10)}, \dots, D_{1(1N)}, D_{1(2N)}, \dots, D_{1(pq)}, \dots, D_{1(N0)}, D_{1(N1)}, \dots, D_{1(NN)})^T$$

$$D_{1(p,q)} = \frac{(-k_x k_y)}{r_{su,j}^2} [\exp(-ik_x a) \cos(p\pi) - 1] [\exp(-ik_y b) \cos(q\pi) - 1] \left[k_x^2 - \left(\frac{p\pi}{a}\right)^2 \right] \left[k_y^2 - \left(\frac{q\pi}{b}\right)^2 \right] \quad (A-2)$$

c. $[H]=[H(\mathbf{l}_x, \mathbf{l}_y, \mathbf{l}_z)]$

$$\overset{\omega}{H} = [H] = (H_{00}, H_{01}, H_{02}, \dots, H_{0N}, H_{10}, \dots, H_{1N}, H_{2N}, \dots, H_{mn}, \dots, H_{N0}, H_{N1}, \dots, H_{NN})^T \quad (A-3)$$

$$H_{mn} = \exp(-ik_{mn}l_z) \cos\left(\frac{m\pi x}{l_x}\right) \cos\left(\frac{n\pi y}{l_y}\right) \cos[k_{mn}(l_z - z)]$$

d. **Modal Numbers:**

m, n, p, q=0, 1, 2, \dots, \infty. Since the investigation in this paper is focused on the low frequency traffic noise, then m, n, p, and q are approximately cut off as 0. 1. 2. \dots, N.