

Frequency range extension of sound diffusers by means of multiobjective optimization

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ABSTRACT

The frequency range of a Schroeder sound diffuser is determined by its dimensions, depth of the wells and total width. Several authors have studied the relationship between the diffuser dimensions and the lowest and highest frequency with significant diffusion. In this paper we have verified the validity of previous works. Furthermore, we have extended the working frequency range to lower frequencies by means of a multiobjective optimization. Preliminary results reveal that the frequency range can be extended to lower frequencies, particularly if different widths are defined for each well.

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1. INTRODUCTION

Sound diffusers are widely used in room acoustics in order to reduce echoes and focalizations and to reduce the effect of low frequency modes and as a result improving the diffuseness of the sound field. They can be defined as devices on which the sound is reflected in a non-specular way, in other words, Snell's law is not satisfied. The first ones were proposed by Schroeder in 1975 [1] and consist of a set of wells with different depths that come to modify the phase of sound. Due to this, they are known as Schroeder or phase diffusers. When the variation of the depth of the wells is only in one direction, the resultant diffusers are called 1D Schroeder diffusers. Among the different types of Schroeder diffusers, the most popular is the 7 wells QR (quadratic residue) diffuser. In this study we will focus in this particular case. Figure 1 illustrates this sound diffuser.

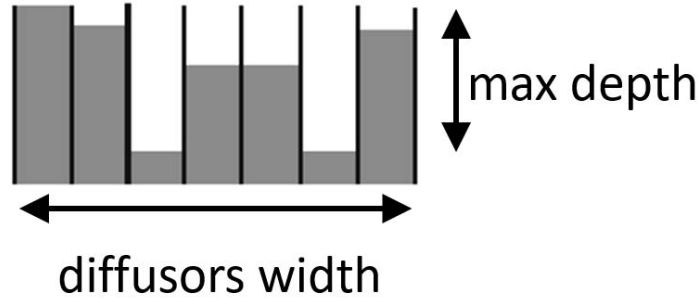


Figure 1. Section of a Quadratic Residue (QR) diffuser of 7 wells.

The quantification of the performance of such a device can be done following two different strategies, both standardized by ISO (International Organization for Standardization) [2-3]. In this work we will follow the standard ISO 17497-2 2012 [3], based on the measurement of the reflected sound pressure over a range of angles, between -90° and 90° in steps of 5° (37 measurements in total). For this purpose a microphone is sequentially positioned along a semi-circumference centered in the middle point of the test sample, which is composed by an array of at least three diffusers (with seven wells each one). The original signal has to be windowed in order to separate the reflected sound from the direct sound. The parameter measured using this technique is known as the diffusion coefficient:

$$d'_j = \frac{(\sum_{i=1}^n p_{ij}^2)^2 - \sum_{i=1}^n (p_{ij}^2)^2}{(n-1) \sum_{i=1}^n (p_{ij}^2)^2} \quad (1)$$

where d'_j is the diffusion coefficient for the j -th one-third octave band considered, p_{ij} is the reflected sound pressure for the j -th one-third octave band considered at the i -th measurement position, and n is the number of measurement positions ($n = 37$). This diffusion coefficient has to be averaged for different incidence angles (in our case 3). To normalize this diffusion coefficient from zero to one, it is compared with that of a flat surface. The purpose of normalization is to remove edge diffraction scattering effects due to the limited size of the sample under analysis. The normalized diffusion coefficient, d_j , for the j -th one-third octave band considered, is defined as:

$$d_j = \frac{d'_j - d_{j,ref}}{1 - d_{j,ref}} \quad (2)$$

where $d_{j,ref}$ is the diffusion coefficient of a flat panel for the j -th one-third octave band considered. As a result, d_j is equal to zero for all frequencies in the case of a flat surface.

All these coefficients require tedious measurements in an anechoic environment unless numerical methods are available. In particular FDTD is a well-established method that can be used for this purpose [4]. Figure 2 illustrates the simulation scheme used for this paper. Several techniques have been used in order to obtain the far field reflected by the sound diffuser (NFFFT: near field to far field transformation) and to remove the incident sound from the simulations (TFSF: total field – scattered field formulation). Further details of the simulations can be found at [4].

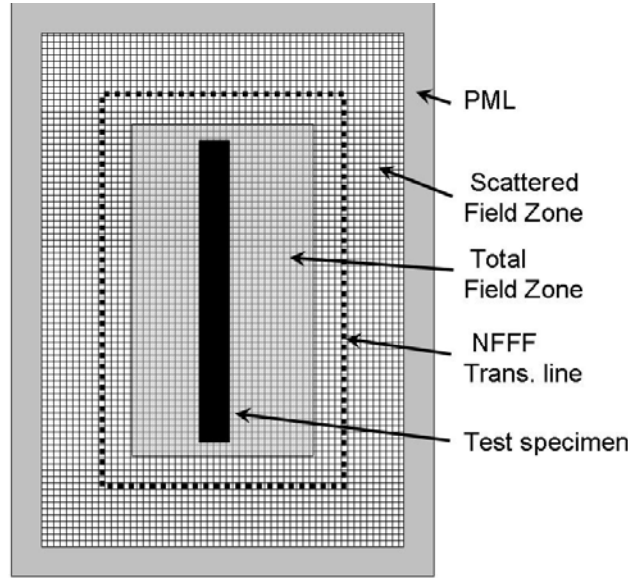


Figure 2. FDTD Simulation Scheme (reference flat panel). The figure shows the two simulation zones (total field zone and scattered field zone). It is as well illustrated the measurement points used to transform to far field (see text). The PML is an absorbing termination to simulate anechoic environment.

In a QR diffuser the depths of the wells are calculated by the following equation [5]:

$$S_n = n^2 \text{ mod } N \quad (3)$$

where N is a prime number on which the sequence is based (7 in this case) and n is the index to element S_n of the sequence. In our case (7 wells) $S_n = [0 \ 1 \ 4 \ 2 \ 2 \ 4 \ 1]$, generated from $n = [0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6]$. The depths of each well is calculated from the sequence as follows.

$$D_n = \frac{S_n c}{2 N f_0} \quad (4)$$

where D_n is the depth of the n -th well (we use here capital letters to avoid confusion with the diffusion coefficient), c is the speed of sound and f_0 is the so called design frequency. For frequencies below f_0 the diffuser tends to behave as a flat surface causing 0 diffusion. As a result, the lowest frequency where significant diffusion is achieved, $f_{\min D}$, (D stands for the effect of the depth of the diffuser) coincides with the design frequency, i.e.:

$$f_{\min D} = f_0 = \frac{S_n c}{2 N D_n} \quad (5)$$

If the previous equation is particularized to the deepest well it reads as follows ($S_{n \max}=4$)

$$f_{\min D} = f_0 = \frac{97.43}{D_{n \max}} \quad (6)$$

where $D_{n \max}$ is the maximum depth of the wells, in other words, the diffusers depth.

Up to this point, the effect of the limited width of the diffusers has not been considered. In reference [6] this was considered for the first time. According to that paper, there is an additional limit that can increase f_{\min} :

$$f_{\min L} = \frac{c}{L} \quad (7)$$

where L is the total width of the diffuser.

In the next section we will consider the accuracy of equations 6 and 7 to evaluate the lowest frequency at which the diffuser is efficient. Without loss of generality, we will consider that a value of the diffusion coefficient larger than 0.35 corresponds to significative efficiency of the diffuser.

2. RESULTS

We have performed a systematic calculation of the effect of the depth and the width of the diffuser. For convenience the maximum depth considered has been 0.34 m (the wavelength at 1kHz) and the width has been limited to 1 m. Next figure illustrates the performance of two QR (7 wells) diffusers with different widths (1 m and 0.6 m) obtained with a FDTD simulation as a function of its maximum depth. Generally speaking, the thinner diffuser has lower values of the diffusion coefficient. However, the lowest frequency, f_{\min} , seems to have very similar behaviour.

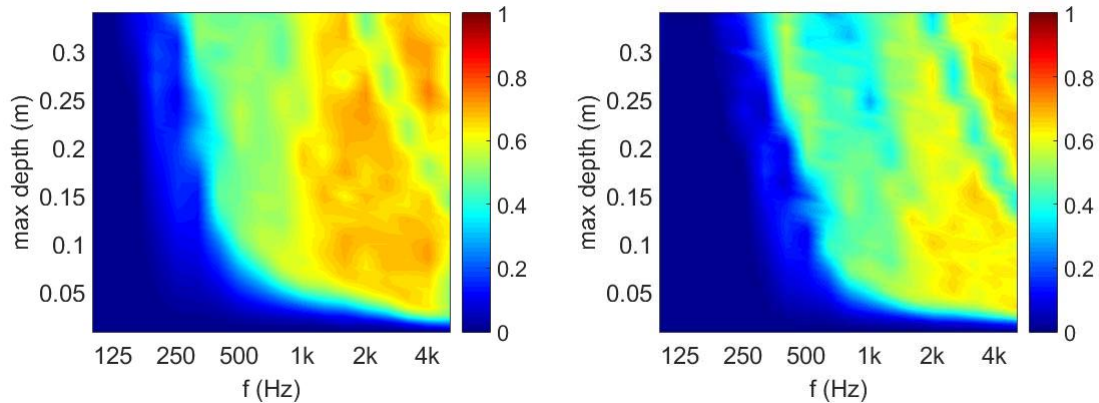


Figure 3. Normalized diffusion coefficient for two different 7 wells quadratic residue diffusers. X axis: frequency (Hz), Y axis: total depth of the diffuser. Left plot corresponds to a diffuser width of 1 meter and right plot to a 0.6 m one.

In order to better study the lowest frequency, Figure 4 illustrates f_{\min} for both cases. To obtain this plot we have found, for each possible depth between the limits (0 to 34 cm), the lowest frequency where significative diffusion is achieved. This implies to set an arbitrary limit for the diffusion coefficient to be considered as significative. Given that the maximum value of the diffusion coefficient is about 0.7, assuming a limit of 0.35 (i.e, 50% of the max) seems to make sense.

So, Figure 4 illustrates f_{min} obtained from the simulations together with the theoretical limits commented above (equations 6 and 7). We can conclude that the limitation due to the width of the diffuser is no relevant and actually in both cases f_{min} follows the theoretical value due to the depth effect, i.e., $f_{min D}$. However this is not true for shallow diffusers, when the maximum depth is lower than 0.2 m.

It is quite remarkable that there is a particular range of values of the maximum depth for which f_{min} is about one octave bellow $f_{min D}$ (see for instance the case of a 1m wide diffuser for a maximum depth below 0.15 m). Actually, this was already pointed out by Schroeder in [7].

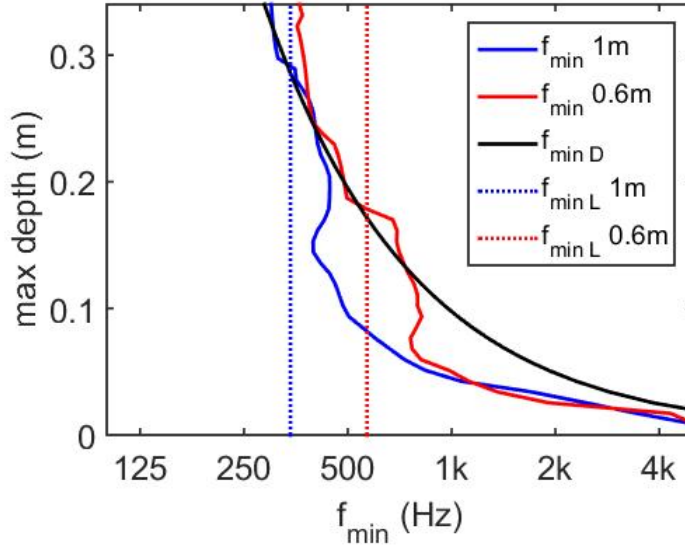


Figure 4. Maximum depth vs f_{min} for two different values of the width (1m (blue) and 0.6m (red)), Continuous lines. Theoretical f_{min} due to the limited depth of the diffuser (6), black line. Theoretical f_{min} due to the limited width of the diffusers (7), dotted lines.

Taking into account the differences between the numerical results for f_{min} and the theoretical predictions, we propose a simple expression to describe f_{min} , which describes reasonably well the observed numerical results, namely:

$$f_{min F} = \frac{f_{min D}}{1 + e^{-\left(\frac{Dn_{max}}{W}\right)^2}} = \frac{f_0}{1 + e^{-\left(\frac{Dn_{max}}{W}\right)^2}} = \frac{\frac{S_n c}{2 N D n}}{1 + e^{-\left(\frac{Dn_{max}}{W}\right)^2}} \quad (8)$$

where W is the width of each well, i.e., $W=L/N$. This fit is plotted in Figure 5. As can be seen, f_{min} follows roughly this qualitative fit. However, the study has to be extended to more cases (different sequences, larger N , and so on), but this is beyond the scope of the present paper.

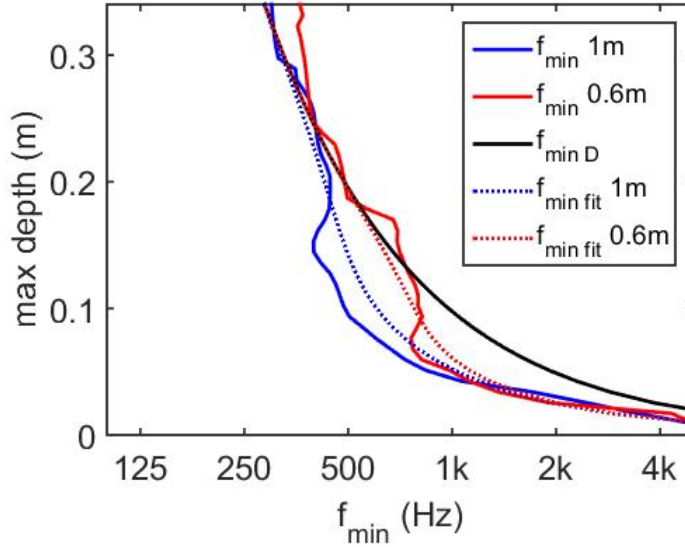


Figure 5. Maximum depth vs f_{min} for two different values of the width (1m (blue) and 0.6m (red)), Continuous lines. Theoretical f_{min} due to the limited depth of the diffuser (6), black line. Fits of f_{min} following Equation 8, dotted lines.

3. OPTIMIZATION

In order to study if the low frequency limit can be overcome, we have performed an optimization. Given that such an optimization implies two parameters, f_{min} and $D_{n,max}$, we have used a multi-objective algorithm, in particular an open multi-objective evolutionary algorithm known as *ev-MOGA* [8]. Further details of the optimization algorithm can be found at [9].

Two different optimizations were carried out. In the first one the widths of the seven wells were fixed to 1/7 the total width of the diffuser. In the second one, the algorithm allowed the wells width to vary between a minimum and a maximum value. For convenience the minimum value was fixed to 4 cm, and the maximum value was set to 76 cm. The first step is to define a gene codification for the diffusers. The possible candidates are encoded by a set of seven genes that represent the normalized depth of each well and, and if applicable, seven genes that represent the normalized width of each well.

The next step is to define cost functions, in other words, the parameters that are to be minimized. As commented above the two parameters are f_{min} and $D_{n,max}$. The second one is trivial but the first one can be problematic. In previous works we observed that if this cost parameter was defined as the lowest frequency for which diffusion was significant, the optimized individuals had a diffusion coefficient that oscillated widely, even falling below the threshold. Therefore, in this work we have defined this cost parameter as the intersection of the diffusion coefficient with the threshold value for the highest frequency within the usual range of study in room acoustics (125 and 4000 Hz octaves). As a result, the optimized individuals will be effective from f_{min} to $4000 \sqrt{2}$ (higher limit of the 4KHz octave band).

A set of 10000 possible individuals generated randomly were introduced in the algorithm as starting point (initial population). Individuals are crossed randomly generating new individuals. New individuals are “measured” according to the cost functions. Eventually any individual can be removed, due to substitution by a new one, from the population if it does not belong to the so called Pareto front, defined as the set of points that are not dominated by any other individual of the population. Dominance

refers to the fact that there is no any other individual with lower values of all the cost functions. After a few generations the Pareto front represent the “best” population that can be found.

It is important to highlight that it is necessary to have enough variability of each gene; in other words, the depth of the wells has to change in a tiny step. This is relatively hard to achieve with FDTD. Due to this we have used a BEM based algorithm instead. Since the classical geometry of a QR diffuser will be modelled, special care must be taken in order to apply the BEM to solve the problem. Indeed, the presence of the walls separating the diffuser’s wells originates very thin surfaces which typically lead the direct BEM formulation to degenerate and lead to unstable equation systems. For this reason, a dual-BEM formulation is used here, in which the direct BEM integral equation (see equation (9)) is complemented by the so-called hypersingular BEM equation (see equation (10)). Details of this formulation can be found in [10], and thus only a general overview is here given regarding the BEM.

The classical boundary integral equation can be derived from the Helmholtz equation in the frequency domain by applying the reciprocity theorem, and in the case of rigid boundaries it can be written as:

$$C p(\mathbf{x}_0, \omega) = - \int_{\Gamma} H(\mathbf{x}, \mathbf{x}_0, \omega, \mathbf{n}) p(\mathbf{x}, \omega) d\Gamma + p_{inc}(\mathbf{x}_0, \mathbf{x}_S, \omega) \quad (9)$$

where G represents the Green’s function for the pressure defined before, and H is its first derivative with respect to the normal direction to the boundary Γ ; similarly, p and q are the pressure and its first derivative in the normal direction to the boundary (\mathbf{n}), at point \mathbf{x} ; $p_{inc}(\mathbf{x}_0, \mathbf{x}_S, \omega)$ represents the effect of a possible acoustic source located at point \mathbf{x}_0 . The factor C equals $1/2$ if $\mathbf{x} \in \Gamma$, and 1 for points not in the boundary but within the domain ($\mathbf{x} \in \Omega$).

The hypersingular boundary integral equation can be derived by taking the first derivative of equation (9) with respect to the surface normal, and thus the required additional integral equation can be expressed as:

$$A p(\mathbf{x}_0, \omega) = - \int_{\Gamma} H'(\mathbf{x}, \mathbf{x}_0, \omega, \mathbf{n}, \mathbf{n}_2) p(\mathbf{x}, \omega) d\Gamma + p'_{inc}(\mathbf{x}_0, \mathbf{x}_S, \omega) \quad (10)$$

The Green's functions G' and H' can be seen as the derivatives of G and H with respect to the normal to the boundary at the loaded point, \mathbf{n}_2 . In this equation, the factor A equals zero for piecewise straight boundary elements.

This formulation is used to analyse each configuration of the diffuser, allowing to compute the sound pressure scattered by the diffuser at any point of the acoustic domain. It is thus called multiple times from within the optimization algorithm, allowing the evaluation of the defined cost function for each individual.

3.1 Optimization results

For the sake of brevity we will present here only the results for a diffuser width of 1 meter. Concerning the optimization with fixed wells width, the results are summarized in Figure 6. It can be seen that optimized diffusers can extend the low frequency working limit about one octave. In other words, the limit proposed by Schroeder in his first papers can be decreased above two octaves.

It is very normal that when doing an optimization the optimal individuals are very similar to each other. The case presented here is no exception. Figure 7 illustrates the 4 families of diffusers that make up the Pareto front. Three clearly differentiated families are observed, the first appears for high depths and is formed by a group of two wells with

very shallow depths and another group of 3 wells with very high depths separated from each other by wells with intermediate depths. In the second family there is only one intermediate deep well. Finally in the last family, which covers the lower depth area, there are basically only two different depths.

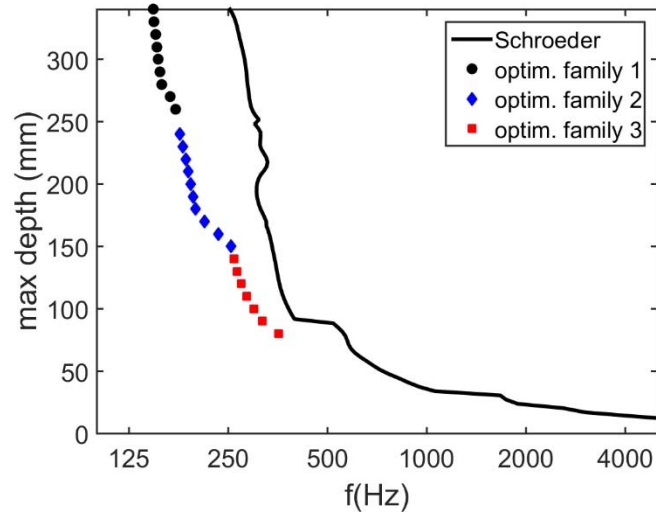


Figure 6. Maximum depth vs f_{min} for QR sound diffuser (width 1m) for different total depths (black continuous line). Families of optimized diffusers: black circles, blue diamonds and red squares.

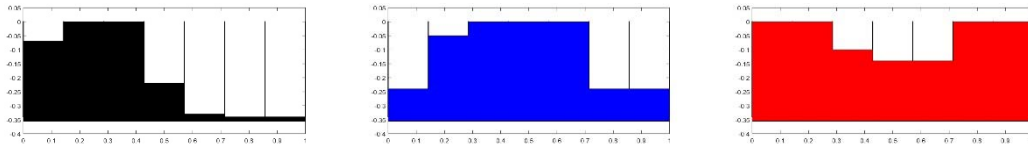


Figure 7. Appearance of the families of optimized diffusers. Left: Family 1, the deepest diffusers. Middle: Family 2. Medium depth. Right: Family 3. Less deep.

Finally Figure 8 shows the preliminary results of the optimization in which the width of the wells is allowed to vary. It can be seen that the improvement is almost insignificant in the deep zone. However, the improvement is not negligible for intermediate depths.

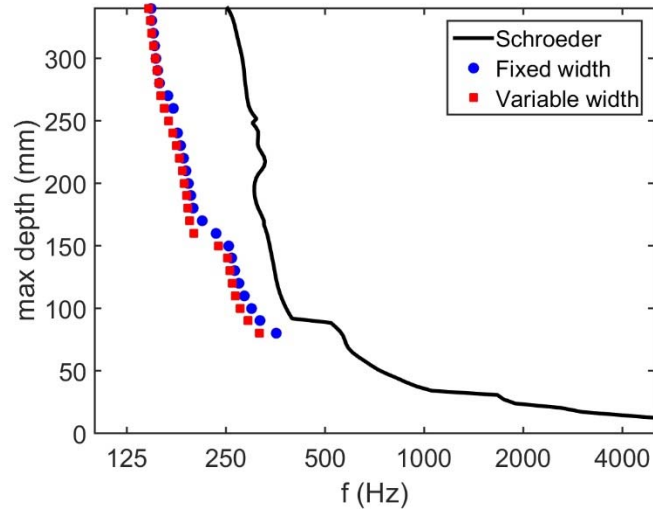


Figure 8. Maximum depth vs f_{min} for QR sound diffuser (width 1m) for different total depths (black continuous line). Optimized diffusers with fixed well widths (blue circles). Optimized diffusers with variable well widths (red squares).

4. CONCLUSIONS

In conclusion, we have shown that the equations commonly used to predict the lower limit of the frequency range of a diffuser can overestimate it up to one octave in particular cases. Furthermore, we have proposed a new equation for the estimation of the lower limit with better agreement with the numerical results. Additionally we have studied the possibility of extending the frequency range of a diffuser for low frequencies by means of a multi-objective genetic algorithm, showing that the low frequency limit can be extended about one octave in comparison with Schroeder diffusers.

5- ACKNOWLEDGEMENT

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