

Sound power minimization of a railway wheel by means of a modal-based geometric optimization technique

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ABSTRACT

The development of computationally fast optimization procedures for railway wheel noise mitigation is crucial in industry and research progress. Within this scope, two main geometric optimization methodologies have been implemented: one purely based on the modal properties of the wheel and the other focused on the explicit sound power calculation as the objective function of the optimization procedure. The former is based on shifting the natural frequencies of those modes which are most relevant for rolling noise generation to a frequency region with lower amplitudes of the excitation contact force. The second approach is based on the calculation of the radiated sound power and provides both a more robust method and a tool for the validation of the main assumptions of the natural frequency driven optimization, which is expected to achieve silent wheel designs with the advantage of a potentially lower computational cost and complexity. In all cases, a high-cycle fatigue analysis for assuring structural feasibility of the candidates and modal analysis is carried out using the Finite Element Method (FEM).

Keywords: Railway wheel, Geometric optimization, Genetic Algorithm, Sound power level

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1. INTRODUCTION

Rolling noise emitted by railway wheels affects human health and restricts railway coverage in the proximities of urban areas [1–3]. For the usual vehicle speed conditions in such locations, rolling noise is predominant over the rest of noise sources from the vehicle/track system [4]. Several components are involved in rolling noise phenomenon, namely wheels, rails, sleepers, etc.

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This works presents an innovative procedure for the minimization of railway wheel rolling noise through the geometric optimization of its cross-section. The proposed methodology is based on the reallocation of the most relevant mode shapes outside the predominant excitation range through a Natural Frequency (NF) maximization strategy, given the lower amplitude content of usual rolling noise excitation signals in the high frequency range [4]. In fact, it is observed in literature how certain wheel cross-sectional geometric features associated with higher wheel rigidity are also related with an overall lower noise radiation levels [4,5].

Previous similar optimization approaches in the field [6, 7] consider an objective function which implies deriving the acoustic power radiation of the wheel. Given the large amount of objective function evaluations required in an optimization process, previous works implement computationally efficient acoustic models, such as the TWINS commercial software. These make important simplifications and have some limitations. More accurate approaches, such as the Boundary Element Method (BEM) [8] are of difficult application for an optimization procedure given their high computational cost. Instead, the objective function implemented in this work is solely based on the wheel NF obtained through a modal analysis, thus avoiding the associated computational complexity of deriving noise radiation and the related model assumptions.

A global optimization technique, namely a Genetic Algorithm (GA), is considered as optimization tool. Moreover, a series of objective function modifying terms are applied in order increase the effectiveness of the method, which take into account the specific excitation force being simulated and modal properties of the wheel. In order to validate the results obtained by the NF-based methodology, the Sound poWer Level (SWL) of the best found solution is evaluated and compared with the best design obtained by a SWL-based methodology, this is, by using an objective function constructed through the computation of the overall SWL for each candidate.

The rest of the document is organized as follows: firstly, the optimization methodologies are presented. Secondly, the results obtained by the NF-based procedure are described and compared with those obtained by the SWL-based method. Finally, a concluding discussion is presented.

2. METHODOLOGY

The main aspects of the model presented in this work are the optimization procedure for both NF-based and SWL-based approaches, and the geometric parametrization of the railway wheel transversal section [7].

2.1. Optimization procedure

The optimization algorithm (see Fig. 1) used in the methodologies explained in the present work proceeds as follows: first, the variables that define the candidate to be simulated \bar{x}_i , are selected from a design space set \bar{x}_0 . The next step is to perform a high-cycle fatigue analysis of the candidate with the aim of determining its structural feasibility. This structural analysis is carried out using a FEM model according to the standard [9] and considers as acceptable only those candidates whose critical stress difference $\Delta \sigma_c$ is lower than the admissible value ΔA_m . In such case, a modal analysis is performed in order to obtain the data required to build the corresponding objective function. Such analysis is configured by the maximum frequency f_m up to which modes

are obtained, the maximum element size h of the FE mesh, and the number of Fourier terms n_f used in the axisymmetric computation, among others. From the obtained modal data (the mode shapes matrix Φ and the NF, $\overline{\omega}$), the acoustically most relevant mode shapes and their mean natural frequencies (Φ_{ID} and $\hat{\omega}_{ID}$, respectively) are selected using a modal identification routine, classifying mode shapes through their nodal diameters n and nodal circumferences m [4]. Using these data, the OF would be evaluated as described in the subsequent sections 2.2 and 2.3. After each GA generation, the convergence criteria fulfilment is checked, which means that the relative change in the objective function is lower than a tolerance tol_m ($tol_m = 10^{-6}$ in this work). If convergence is met, the optimization will end returning the Best Found Solution (BFS) \overline{x}^* . Otherwise, the objective function value Obj will be used to feed the GA, which will generate a new set of candidates in the search of an optimal one.



Figure 1: Block diagram illustrating the main components of the optimization procedure.

2.2. NF based methodology

The goal of this methodology is to maximize the NF of the vibration modes, shifting these NF to a region with less excitation frequency content and, at the same time, preventing undesired modeshape re-allocations into more excited frequency regions. More specifically, the objective function Obj of this optimization approach is defined as,

$$Obj = Obj_b \cdot m_e \cdot m_p = \left[\frac{1}{(\hat{\omega}_{m,ID})^k}\right] \cdot \left[\frac{-\tanh\left(\frac{\hat{\omega}_{m,ID} - f_0}{s} + 1\right)}{2}\right] \cdot \left[1 + \left|\frac{\hat{\omega}_{p,ID} - \hat{\omega}_{p,ID}^0}{\hat{\omega}_{p,ID}}\right|\right]$$
(1)

where $Ob j_b$ is a base objective function, built with the mean NF $\hat{\omega}_{m,ID}$ of those modes whose contribution is more relevant for the generation of sound radiation. $Ob j_b$ is combined with two modifying masks, the so-called *excitation* and *penalization masks*.

The aforementioned *excitation* mask m_e is implemented in the form of an hyperbolic tangent function as shown in Eq. 1, and takes into account the frequency content of the combined excitation roughness under consideration, which is defined according to the

standard [9]. It favours the re-allocation of mode shapes towards the region considered as having less significant amplitude content, determined by f_0 . The modal up-shifts that cross f_0 will be highly valued by the GA, opposite to those which do not cross this value. As a result of this fitness gap between candidates, whose magnitude would depend on parameter *s*, the GA will focus its resources on those candidates whose selected $\hat{\omega}_m$ cross f_0 . For further understanding of this mask behavior, a representation of m_e for different values of *s* is represented in Fig. 2. The parameters selected for the optimization procedure are those used in [10].

The *penalization mask* m_p , controls the shift of mode shapes whose NF increase may re-allocate them in a point of higher excitation amplitude in the frequency domain with respect to their state in the reference design. Such a phenomenon may happen when the excitation amplitude is not monotonically decreasing with the frequency. In order to prevent this, the up-shift of the average of the NF of a set of modes selected $\hat{\omega}_{p,ID}$ is penalized, taking as reference their initial mean value $\hat{\omega}_{p,ID}^0$.



Figure 2: m_e function times $Ob j_b$ for different values of s, $f_0 = 1388$ Hz and k=0.05.

2.3. SWL methodology

The main goal of the SWL methodology is directly minimizing the SWL of the wheel. In this case the objective function used consists of the summation of the radiated sound power in each frequency band.

For the SWL calculation model [4], a contact force between the rail and wheel is generated as

$$\{F_c\} = \left[\alpha_{sys}\right]^{-1} \cdot \{r\}$$
⁽²⁾

where F_c is the contact force; α_{sys} represents the combined receptances of the system conformed by the rail, the wheel and the contact between them; and r is the combined roughness considered for the optimization problem. For the derivation of α_{sys} , a Timoshenko beam model is used for the receptance of the rail, the contact is modeled with a contact spring in the radial direction and the wheel receptance is defined as

$$\alpha_{jk}^{Wheel} = \sum_{l} \frac{\Psi_{lj} \Psi_{lk}}{m_{l}(\omega_{l}^{2} - \omega^{2} + 2i\xi_{l}\omega_{l}\omega)},$$
(3)

 $\Psi_{lj,k}$ being the modal amplitude of mode *l* in the position *j* where velocity is calculated or *k* where the force is applied, m_l the corresponding modal mass, ω_l the natural frequency of the mode and ξ_l the modal damping. As in the NF case explained in section 2.2, the

roughness is chosen for cast iron brake blocks and a contact filter is applied according to the standard [9].

Once F_c is generated, it is possible to calculate the velocity response on the surface of the wheel through the wheel mobility to finally obtain the sound power of the wheel W through

$$W = \sum_{l} \sigma_{l} \rho_{0} c_{0} S \langle \overline{v^{2}} \rangle_{l}, \qquad (4)$$

where σ_l is the radiation efficiency of each modeshape *l*, which is computed according to literature [11]; ρ_0 and c_0 represent the air density and the speed of sound respectively; *S* is the area of the radiating surface and $\langle \overline{v^2} \rangle_l$ is the root-mean squared velocity of each mode *l* on the surface area. Then, to obtain the SWL, *W* is expressed in dB(A) to include the effects of how human beings perceive sounds. Finally, the SWL of the summation of the calculated sound power in each frequency band is used to generate the objective function for the SWL-based approach:

$$Obj = SWL_{\Sigma} = 10\log_{10}(\sum_{fband} 10^{\frac{SWL_{fband}}{10}}).$$
(5)

3. GEOMETRY PARAMETRIZATION

Regarding the generation of the FE model used within the optimization algorithm, a geometric parametrization of the transversal section of the wheel, reflected in Figure 3 was implemented. The FE mesh is composed by general axisymmetric elements, used in order to reduce the computational cost associated with the modal analysis.

This parametrization is defined by four design variables found in literature [7,11] to be highly influential on the acoustic radiation: wheel radius x_1 , fillet radii x_2 , web thickness x_3 and web offset x_4 . It should be stated that, although x_1 and x_4 are directly related to its corresponding design variable, x_2 and x_3 are defined in terms of a scalar multiplier to a reference value. Therefore, the actual value of the fillet radii or the web thickness will result from the addition of its reference values to a base value multiplied by x_2 and x_3 respectively. The reference wheel design considers the typical dimensions for a railway wheel and the design domain is selected according to manufacturing and compatibility constraints, defined by the design boundaries. These design domain details are shown in Table 1.



Figure 3: Geometric design variables and an explanatory scheme (shaded) with reduced x_1, x_2, x_3 and increased x_4 parameters.

	Design variables						
	<i>x</i> ₁ [m]	<i>x</i> ₂	<i>x</i> ₃	<i>x</i> ₄ [m]			
Reference	0.4966	0.8221	0.0681	0.0300			
Lower Boundary	0.4	-0.1	0.7	-0.27			
Upper Boundary	0.5	0.1	0.93	0.27			

Table 1: Design domain for the optimization methodologies.

4. **RESULTS**

A wheel shape optimization of the transversal section of a railway wheel is performed where two different approaches have been followed: the maximization of the natural frequencies of those mode shapes with a higher contribution to the sound radiation (NFbased) and a direct minimization of the SWL of the sum of sound power calculated for the wheels in each frequency band (SWL-based). In addition, structural tests are implemented to assure that the resulting wheel designs meet the required structural criteria. In all cases, the population size is 50 candidates.

The main results for both methodologies are shown in Table 2. Both approaches show a reduction in the total SWL (SWL_{Σ}) with respect to the reference wheel, with 2.48 and 4.87 dB(A) for the NF-based and SWL-based optimization, respectively. Remarkably, the NF-based procedure converged at a significantly less number of generations than in the SWL approach. This would confirm one of the main assumptions of the work, i.e., that the NF methodology allows for a reduction of the SWL while greatly decreasing the computational requirements of the optimization procedure compared with other more complex approaches based on calculating the actual SWL of each candidate. This advantage would be enhanced if a more accurate and computationally expensive SWL solver was used, such as BEM.

	Design variables					
	<i>x</i> ₁ [m]	x_2	<i>x</i> ₃	<i>x</i> ₄ [m]	$SWL_{\Sigma} dB(A)$	Generations
Reference	0.4966	0.8221	0.0681	0.0300	116.82	Ø
NF-based	0.4000	0.9037	0.0926	-0.0083	114.34	75
SWL-based	0.4031	0.7948	0.0969	-0.0089	111.95	176

Table 2: Optimization parameters for each BFS compared with the reference wheel.

As shown in Table 2 and Figure 4, the resulting geometric parameters are similar for both methodologies and agree with what it would be expected from the literature [4, 5]. In both cases, with respect to the reference case, the wheel radius has decreased, the web thickness has increased and the resulting web shape is straight in both cases, reducing the radial-axial vibration coupling.



Figure 4: (a) BFS with the NF-based methodology (blue, continuous line) and reference design (discontinous line). (b) BFS with the SWL-based methodology (orange, continuous line) and reference design (discontinous line). (c) NF-based and SWL-based BFS comparison.

In the BFS of the NF-based optimization, the NF of the mode shapes of the wheel are shifted to a higher frequency region where the amplitude of the corrugation is lower as depicted in Fig. 5. In this work, the vibration modes selected for NF maximization are the most acoustically relevant [4], namely n = 2, 3, 4; m = 1, thus $\hat{\omega}_{m,ID}$ is obtained using the NF of these mode shapes. The results shown in Fig. 5 reflect the expected behaviour of the optimization using the base objective function in combination with excitation mask m_e . Furthermore, it is observed that the application of m_e also improves the convergence speed of the optimization. In regard to the *penalization mask* m_p , the n = 2; m = 0modeshape - or (2, 0) - was targeted for its re-allocation constraint, since it is located in a local minimum of the corrugation spectrum, at approximately 300 Hz. As shown in Fig. 5 the (2, 0) modeshape was shifted to a excitation peak near 400 Hz after optimization. It is thought that this negative result is due to a high coupling between the vibration modes, suggesting that the m_p term has a neglible effect.

5. CONCLUSIONS

A geometric optimization procedure of a railway wheel is performed following two different approaches, one where the objective function is built from the NF of the vibration modes (NF-based) and another built with the total SWL (SWL-based). In all cases, structural criteria and roughness spectrum effects are taken into account.



Figure 5: (a) Frequency shift of the selected vibration modes from the reference wheel (black) to the BFS (blue) through the NF-based methodology. (b) frequency content of the roughness used in the excitation.

Results reflect that in both approaches a relevant SWL reduction is accomplished. The NF-based methodology, although not as effective in terms of SWL minimization as the SWL-based approach, can be significantly more efficient, reducing the computational complexity required to evaluate the objective function and increasing the convergence speed. In this work, although the computational time required for the evaluation of the objective function for a single candidate is similar for both approaches, since a simplified SWL model is used, the global time required has been considerable lower in the NF-based optimization due to its faster convergence. Additionally, if a more complex SWL simulation tool were used, such as BEM, the difference in computational cost of both methodologies will potentially be much higher.

The noise reduction phenomenon in the NF-based methodology is explained through the observed frequency re-allocation of the acoustically important vibration modes, shifted to a region where excitation amplitude is less relevant. Interestingly, the indirect NF-based approach leads to a similar wheel design than the direct SWL-based approach, obtaining geometric parameters consistent with what it is observed in the literature. In summary, it is suggested that the NF-based optimization might be a suitable methodology when computational resources are limited.

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