Swarm of robot attacking an acoustic source: detection and trapping.

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ABSTRACT

Source identification in a complex environment has a remarkable interest in acoustic and noise-engineering as well as in defence applications. The detection of mobile sources through a swarm of drones used as a set of microphones carriers is proposed in the present paper formulating a new theory to solve the problem. A set of N carriers of sensors, each of them equipped by its own dynamics, moves in the environment and can detect the local acoustic field as an effect of the noise emission of unknown mobile sound-sources. Sound received from the microphones on the carriers is the only detectable signal in the field and the only trace the noise-target releases into the environment. The CAI (Centralized Artificial Intelligence) can use the information coming from the microphones to localize in the best way the source. The swarm pattern noise-source searching is piloted by the CAI that controls the swarm operation and suggests the best re-localization of the agents and thus of the microphones at each time step, with the aim of localizing and trapping the noise-source in an optimal fashion. The mission is completed when the drones localize and reach the noise target.

Keywords: Identification, Swarm Dynamics, Control.
I-INCE Classification of Subject Number: 76

1. INTRODUCTION

In many recent applications it is of great importance the ability of identification of some environment characteristics. Examples are found in acoustics, elastic structures or environmental pollution.

A moving acoustic source injects into the air or water wave energy that propagates and can be detected by suitable sensors (microphones or hydrophones). For example, starting from the signals perceived by the microphones, one can be interested in the reconstruction of the acoustic field [1-3]. On this basis it is possible to make a preventive identification of the source useful in many engineering applications such as damage identification [4], mechanical structures prevention subjected to unscheduled wear due to stick-and-slip contact phenomena [5] or general machinery malfunctions. Analogously, a source of pollutant can generate, by diffusion, a field of concentration in the air environment. Pollutant sensors can detect the characteristics of concentration and possibly can predict position and emission rate of the source.

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The present is a problem of identification, with the peculiarity that the sensors can be displaced into the field and their motion can be suitably used to enhance the detection of its characteristics. Moreover, as it happens in real problem, the sensors cannot be instantaneously moved at given desired placements, rather their repositioning is subjected to the dynamics of the carriers where they are mounted on board. The system artificial intelligence can only act on suitable commands and actuators to drive the sensors positioning, but always subjected to the carries dynamics [6-7]. The dynamics of the problem is rather complex in that its state dynamics involves the carrier dynamics, the acoustic field propagation, the sensors acquisition, and for each elements of the chain, polluting noise is present.

2. SWARM IDENTIFICATION AND TRAPPING: PROBLEM STATEMENT

The statement of the problem we desire to attack here is formulated as follow.
A set of moving sensors $\{s_1, s_2, \dots, s_N\}$ can explore a space region R within which a scalar field $\Phi\left(\underline{\xi}, t\right)$ exists. Each of the sensors is installed on board of the *i-th* carrier and moves autonomously with an associated state variable $\underline{x}_i$. The dynamics of the single carrier is in the form:

$$\underline{\dot{x}}_i = \underline{f}_i(\underline{x}_i, \underline{u}_i) \qquad (1)$$

The set of the carriers obeys the differential equations:

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u}) + \underline{n}_x \qquad (2)$$

where $\underline{f}$ represents the characteristic dynamics of the swarm, $\underline{x}^T = [\underline{x}_1, \underline{x}_2, \dots, \underline{x}_N]$, $\underline{f}^T = \left[\underline{f}_1, \underline{f}_2, \dots, \underline{f}_N\right]$, and $\underline{n}_x$ is the residual unmoddelled part of the carriers dynamics, and finally $\underline{u}^T = [\underline{u}_1, \underline{u}_2, \dots, \underline{u}_N]$ is the vector collecting their controls.

The on board sensors of the carriers detect signals in R of points $\underline{\xi}$, produced by the presence of the field $\Phi\left(\underline{\xi}, t\right)$. The signal generated by the *i-th* sensor is a scalar s_i related to the field Φ by the relationship:

$$s_i(t) = g_i\, \Phi(\underline{x}_i, t) + n_{s_i} \qquad (3)$$

where the field Φ is evaluated at the point $\underline{\xi}_i$, part of the state vector $\underline{x}_i$ of the carrier ($\underline{x}_i = \left[\underline{\xi}_i, \underline{v}_i\right]$, where $\underline{v}_i$ is the velocity of the *i-th* carrier).

The identification of the observed field through the sensors set $\{S_1, S_2, \dots, S_N\}$ is much more effective if a model of the field Φ is available.

For example, for an acoustic source moving in R, suppose Φ is the acoustic pressure, then we know the field obeys the equation;

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = Q(t)\delta\left[\underline{\xi} - \underline{\xi}_Q(t)\right] \qquad (4)$$

where the entire field is originated by the two functions of the time $Q(t)$ and $\underline{\xi}_Q(t)$, i.e. from the intensity and the location of the source, respectively. Therefore, the statement of the problem can be set as follows:

<u>Statement</u>: a swarm of N carriers is given, described each by the state variable $\underline{x}_i$, each carrying one sensor S_i generating the signal $s_i(t)$ due to the field $\Phi\left(\underline{\xi},t\right)$. The field Φ obeys a known partial differential equation $D_{\underline{\xi},t}(\Phi)=0$, where $D_{\underline{\xi},t}$ is a differential operator with respect both to the space and the time variables, respectively. $\underline{u}$ represents the control of the swarm. The set of equations is given:

$$\begin{cases} \underline{\dot{x}} = \underline{f}\left(\underline{x},\underline{u},t\right) + \underline{n}_x \\ D_{\underline{\xi},t}(\Phi) = Q(t)\delta\left[\underline{\xi} - \underline{\xi}_Q(t)\right] + n_\Phi \\ s_i(t) = g_i\,\Phi(x_i,t) + n_{s_i} \quad i = 1,2,\dots,N \end{cases} \tag{5}$$

<u>Goal</u>: identify, in the most efficient way, in some sense to be specified later, $Q(t)$ and $\underline{\xi}_Q(t)$, provided we can act on the control $\underline{u}$ and that the signals s_i are acquired.

The request to reach a source is formulated using the optimal control theory by the minimization or maximization of a given cost functions J:

$$J = \min_{\substack{\underline{u}\in U \\ \underline{x}\in X}} \int_0^T E(\underline{x},\underline{u})dt \tag{6}$$

The objective function $E(\underline{x},\underline{u})$ can indicates, for example, the distance or intensity of any source that must be reached by the carrier, satisfying state and control constraints $\underline{x} \in X$, $\underline{u} \in U$ respectively. The problem (Equation 6) is solved introducing the dynamic system constraints (5) through lagrangian multipliers and often a direct feedback control solution is obtained by using variational feedback control (VFC) tecniques [8,9].

To simplify the problem assume a linear dynamics for the swarm of sensor carriers, and consider the case $\Phi \equiv p$ as an acoustic field described by the linear wave equation:

$$\begin{cases} \underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{n}_x \\ s_i = g_i p(x_i,t) + n_{s_i} \\ \nabla^2 p\left(\underline{\xi},t\right) - \frac{1}{c^2}\ddot{p}\left(\underline{\xi},t\right) = Q(t)\delta\left[\underline{\xi} - \underline{\xi}_Q(t)\right] + n_p \end{cases} \tag{7}$$

The problem with respect to the classical approach to the Kalman filter is remarkably different. In fact, in those classical problems s_i is proportional to $\underline{x}$ and the objective of the estimate is $\underline{x}$ and not the measured field p. Moreover, the optimal control problem through $\underline{u}$ is not that of requiring an optimal performance for $\underline{x}$, but for s_i. Moreover, as it appears in this case we need two models for the $\underline{x}$ and p dynamics, respectively.

3. REDUTION TO A KALMAN'S FILTER PROCESS

Let us to expand the 2D pressure field as:

$$p\left(\underline{\xi},t\right) = P_{jk}\left(\underline{\xi}\right)q_{jk}(t) \tag{8}$$

where Einstein's notation for summation is used, and the projection functions $P_{jk}\left(\underline{\xi}\right)$ are known:

$$P_{jk}\left(\underline{\xi}\right) = C_{jk}\sin\left(\frac{j\pi x}{l_x}\right)\sin\left(\frac{k\pi y}{l_y}\right) \tag{9}$$

The wave equation becomes:

$$\nabla^2 P_{jk}q_{jk} - \frac{1}{c^2}P_{jk}\ddot{q}_{jk} = Q\delta\left(\underline{\xi} - \underline{\xi}_Q\right) + n_p \tag{10}$$

Assuming the set P_{jk} is orthonormal, decoupled equations in terms of the principal coordinates are:

$$\ddot{q}_{jk} + \omega_{jk}^2 q_{jk} = QP_{jk}\left(\underline{\xi}_Q\right) + \int_R \; P_{jk}\, n_p d\underline{\xi} \tag{11}$$

i.e.

$$\underline{\ddot{q}} + \underline{\underline{\lambda}}\underline{q} = Q\underline{P}\left(\underline{\xi}_Q\right) + \underline{n}_p \tag{12}$$

For the sensors equation, we have:

$$s_i = g_i P_{jk}\left(\underline{x_i}\right) q_{jk}(t) + n_{s_i} \tag{13}$$

In matrix form this equation is written as:

$$\underline{s} = \underline{\underline{\tilde{P}}}(\underline{x})\underline{q} + \underline{n}_s \tag{14}$$

In general, the dimensions of $\underline{s}$ are much smaller than those of $\underline{q}$ (in fact the acoustic field can ideally contain an infinite number of modes, the sensors number remaining indeed finite) and the matrix:

$$\underline{\underline{\tilde{P}}} = \begin{bmatrix} g_1\underline{P}^T\left(\underline{x}_1\right) \\ g_2\underline{P}^T\left(\underline{x}_2\right) \\ \vdots \\ \vdots \\ g_N\underline{P}^T\left(\underline{x}_N\right) \end{bmatrix} \tag{15}$$

is rectangular, with the number of rows much smaller than the number of coloumns.

Therefore, we can rewrite the equations of the problem as:

$$\begin{cases} \underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{n}_x \\ \underline{\ddot{q}} + \underline{\underline{\lambda}}\underline{q} = \underline{\Psi} + \underline{n}_p \\ \underline{s} = \underline{\underline{\tilde{P}}}(\underline{x})\underline{q} + \underline{n}_s \end{cases} \tag{16}$$

where $\underline{\Psi} = Q\underline{P}\left(\underline{\xi}_Q\right)$ is the vector with the unknowns variables, $Q,\ \xi_{x_Q},\ \xi_{y_Q}$.

Let us reduce to a first order problem introducing $\underline{\dot{q}} = \underline{y}$:

$$
\begin{cases}
\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{n}_x \\
\dot{\underline{y}} = -\underline{\underline{\lambda}}\underline{q} + \underline{\Psi} + \underline{n}_p \\
\dot{\underline{q}} = \underline{y} \\
\underline{s} = \underline{\underline{\tilde{P}}}(\underline{x})\underline{q} + \underline{n}_s
\end{cases}
\tag{17}
$$

The modelling can be reduced to an assembly that includes (i) the carriers dynamics, (ii) the acoustic field model, and (iii) the auxiliary equations. They produce together a unique model of the system, including the carreiers and the environment they explore. The fourth equation is the sensor model. Proceding as for Kalman's filter approach, let us write the assembly of the first three theroretical models:

$$
\underline{r}^T = \left[\underline{x}, \underline{y}, \underline{q}\right]^T \tag{18}
$$

$$
\dot{\underline{r}} = \begin{bmatrix} \underline{\underline{A}} & 0 & 0 \\ 0 & 0 & -\underline{\underline{\lambda}} \\ 0 & I & 0 \end{bmatrix} \underline{r} + \underline{\underline{B'}}\underline{u} + \underline{\Psi'} + \underline{n'} = \underline{\underline{A'}}\,\underline{r} + \underline{\underline{B'}}\underline{u} + \underline{\Psi'} + \underline{n'} \tag{19}
$$

where prime denote augmented matrices. The problem statement becomes:

$$
\begin{cases}
\dot{\underline{r}} = \underline{\underline{A'}}\,\underline{r} + \underline{\underline{B'}}\underline{u} + \underline{\Psi'} + \underline{n'} \\
\underline{s} = \underline{\underline{\tilde{P}}}(\underline{r})\underline{r} + \underline{n}_s
\end{cases}
\tag{20}
$$

This statement exhibits the form of an optimal observation problem. This includes a nonlinearity into the sensors dynamics and the presence of a statistical-deterministic disturbance $\underline{\Psi'} + \underline{n'}$ in the carriers model. However, both of these difficulties can be approached with an extended Kalman filter [10-13], and by the deterministic approach to Kalman filtering. Once the optimal observer $\underline{\hat{r}}$ is determined, the otpimal observer $\underline{\hat{q}}$ of the pressure field principal components can be extracted, and the scalar pressure $\hat{p}\left(\underline{\hat{x}_i}, t\right) = P_{jk}\left(\underline{\hat{x}_i}\right)\hat{q}_{jk}(t)$ at the optimally estimated positioning $\underline{\hat{x}_i}$ is finally determined. The optimal control problem can be approached in the context of LQG that permits to drive the swarm in a way to optimize the acoustic field identification.

The problem of the identification of the source term $\underline{\Psi'}$, that icludes the acoustic intensity and the source location, and its possible tracking proceeds in a separated fashion and uses in principle the equation $\ddot{\underline{\hat{q}}} + \underline{\underline{\lambda}}\underline{\hat{q}} = \underline{\Psi}$ or even properties derived directly from the original wave equation $\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = Q(t)\delta\left[\underline{\xi} - \underline{\xi}_Q(t)\right]$, as it is illustrated in the next section.

4. THE THREE MODULES OF THE CAI OF THE SWARM

The strategy for the source tracking and the definition of the objective function to state an optimal control problem, are still based on the set of the previous investigated equations. The previous paragraph shows the swarm formulation problem is based on the swarm dyanmics, the acoustic dynamics, and the sensor equations.

In general, the CAI we are programming is based on a three-steps strategy. The first has been described in the previous section and is based on the *Reduction to a Kalman Filtering*, named *RKF*. The aim of this phase is that of filtering the background noise effects, both for acoustic extra-soruces present into the environment besides the acoustic target, as well as for the control problem. As a result, the motion of the swarm is piloted in order to percieve as cleaner as possible the acoustic field, driving the drones in the best acousting listening regions. The availability of the optimal observers for the acoustics pressure and for the carriers states, permits to use the acosutic signalling from the source in an optimal fashion. To this observation it is related the second step of the strategy of the CAI of the swarm, named *IOS- identification of the source*. A system based on the *TOA-time of arrival* of the acoustic signals, directly related to the acoustic modelling of the environment, can be triangulated to identify the source position $\underline{\xi}_Q(t)$. Once $\underline{\xi}_Q(t)$, a further guidance process to source track can be activated, the third-step of the CAI, named *ST-Source Tracking*. In this phase the system defines an objective function based on the goal of attacking the source. In this context it is assumed the best strategy of control is that to point the source diminishing the distance from the sensors, hoping this leads to better signal to noise ratio. The *IOS* process is always active, since it does not imply any control action, but it consists of data analysis. Switching between the *RKF* and *ST* is indeed managed by a supervisor on the basis of extra-information provided by the system of sensors. In fact, the two methods cannot be used simulatenously because they imply different controls. An alternating automatic switching between the two is possible, but some indicatiors can be used to promote the switching in an optimal manner. In fact, both the methods have internal errors as a reference of the quality of the identification they are promoting. The *RKF* can at any time compare the sensor measurements and the optimal observers to built up a KPI of the quality of signals on board. The same is true also for the *ST*, since the triangulations can give an estimate with error of the position of the source. Comparative analysis of the errors can switch between the two guidance systems.

The *ST-Source Tracking* is aimed at minimizing:

$$\min_{\substack{\underline{u}\in U \\ \underline{x}\in X}} \int_0^T \sum_{i=1}^N \left(\underline{x}_i - \underline{\xi}_Q\right)^T \left(\underline{x}_i - \underline{\xi}_Q\right) dt \qquad (21)$$

that implies the following objective function is selected:

$$\min_{\substack{\underline{u}\in U \\ \underline{x}\in X}} \frac{1}{2}\int_0^T \left(\sum_{i=1}^N \left(\underline{x}_i - \underline{\xi}_Q\right)^T \left(\underline{x}_i - \underline{\xi}_Q\right) + \underline{u}^T \underline{\underline{R}}\, \underline{u} + \underline{\lambda}^T \left(\underline{\dot{x}} - \underline{\underline{A}}\, \underline{x} - \underline{\underline{B}}\, \underline{u}\right)\right) dt \qquad (22)$$

In this context, based on some data provided by the *RKF* module, we solve a standard LQR problem leads to $\underline{u} = \underline{\underline{K}}\, \underline{x}$, updating $\underline{\xi}_Q$ through the *IOS* module or by solving $\underline{\ddot{\hat{q}}} + \underline{\underline{\lambda}}\underline{\hat{q}} = \underline{\Psi}$. As a result we track the field source and the swarm follows the identified source at the end locking it.

5. THE ACOSUTIC FIELD AND THE CARRIERS MODELLING

Let's consider the 2D-wave equation, which describes the propagation of the acoustic pressure field *p* over the region *R*:

$$\nabla^2 p(x,y) - \frac{1}{c^2}\frac{\partial^2 p(x,y)}{\partial t^2} = Q(t)\delta\left[\underline{\xi}(t) - \underline{\xi}_Q\right] \qquad (23)$$

$p(x, y)$ is the acoustic pressure field along the x and y directions at the time t and c is the speed of sound. At the boundaries Mur's absorbing conditions are applied to avoid reflections and echo phenomena, to minimize the disturbances on the measurements:

$$\begin{cases} \left.\frac{\partial p}{\partial x}\right|_{x=0} = c\left.\frac{\partial p}{\partial t}\right|_{x=0}, & \left.\frac{\partial p}{\partial x}\right|_{x=L_x} = -c\left.\frac{\partial p}{\partial t}\right|_{x=L_x} \\ \left.\frac{\partial p}{\partial y}\right|_{y=0} = c\left.\frac{\partial p}{\partial t}\right|_{y=0}, & \left.\frac{\partial p}{\partial y}\right|_{y=L_y} = -c\left.\frac{\partial p}{\partial t}\right|_{y=L_y} \end{cases} \tag{24}$$

The modeled source is supposed to be harmonic $Q(t) = Asin(\omega t)$, with angular frequency $\omega = 4.71\ rad/s$, located at the point of coordinates $\xi_Q = (6.96, ; 0.90)$.

The wave model is dervied by using a finite difference method in space and time:

$$p_{i,j}^{t+1} = 2p_{i,j}^{t} - p_{i,j}^{t-1} + C^2\left(p_{i+1,j}^{t} + p_{i-1,j}^{t} - 4p_{i,j}^{t} + p_{i,j+1}^{t} + p_{i,j-1}^{t}\right) + \Delta t^2 Q_{i,j}^{t} \tag{25}$$

the indexes i and j refer to the x and y axes, respectively. $C = c\frac{\Delta t}{\Delta x}$, is the Courant-Friedrichs-Lewy condition coefficient, which for the present case has been chosen equal to 0.15. In Figure (1) it is reported a shot of the generated field within the region R.

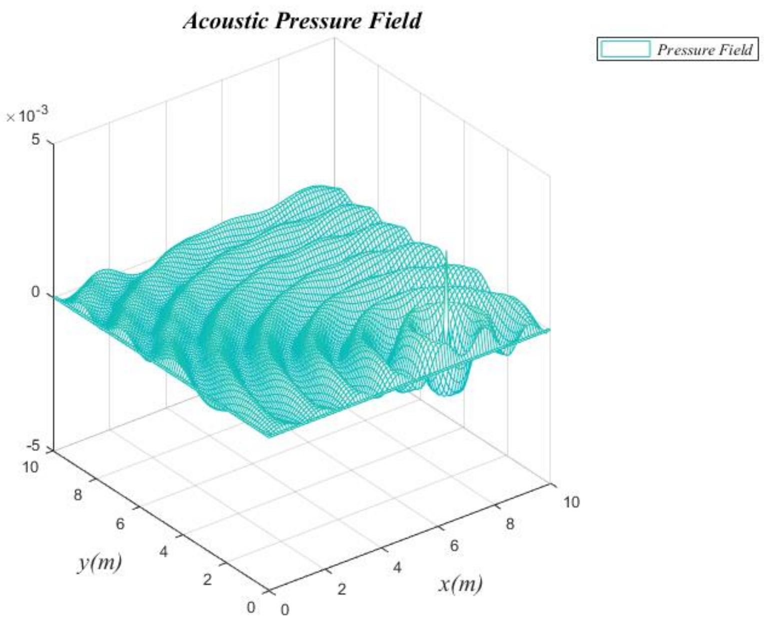

Figure 1 - Acoustic Pressure Field propagating in the region R. Mur's Boundary condition are applied.

Dynamic equations of carriers have been represented through a moving mass system subjected to external forces on xy-plane:

$$\begin{matrix} m\dot{v}_x = F_x \\ m\dot{v}_y = F_y \end{matrix} \tag{26}$$

The state space representation for a single carrier can be expressed as follows:

$$\begin{Bmatrix} \dot{\xi}_x \\ \dot{\xi}_y \\ \dot{v}_x \\ \dot{v}_y \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \xi_x \\ \xi_y \\ v_x \\ v_y \end{Bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \tag{27}$$

where $\xi = (\xi_x, \xi_y)$ and $v = (v_x, v_y)$ are the position and the velocity of the single carrier, respectively, m is the mass of the single carrier and $\underline{u} = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix}$ is the vector of the control. The swarm dynamics can be easily synthesized in the canonical linear time invariant system:

$$\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} \tag{28}$$

6. SOURCE IDENTIFICATION

At every time step, the state vector of the swarm of sensors is considered acquired, i.e. positions and velocities are given through a GPS system, and it is possible to localize the source using only measurements through the *TOA* technique.

This uses the absolute time of arrival of a signal to a certain carrier, departing from a second unknown point (the field source). The distance between the observer and the source can be estimated by the *TOA*. Known the speed of sound *c*, the distance is:

$$r = c * t_{arrival} \tag{29}$$

from which the circumference with radius *r* which identifies the range of possible positions of the unknown point of origin of the signal is:

$$\left(\xi_x - \xi_{x\,carrier}\right)^2 + \left(\xi_y - \xi_{y\,carrier}\right)^2 = r^2 \tag{30}$$

To localize the source, it can be readily shown that it's necessary to use information coming from at least three sensors. Indeed, the intersection of the three obtained circumferences will give a precise estimation of the unknown position, as shown in Figure (2):

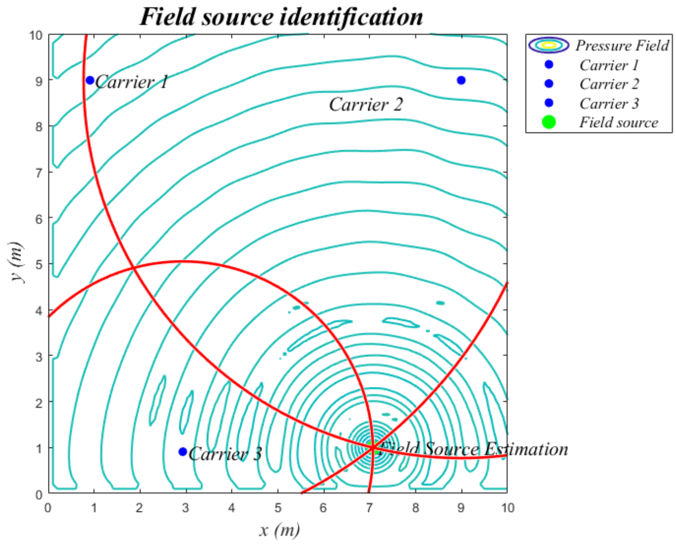

Figure 2 - Identification of the Field source. Intersection of the circumferences identifies the Target point.

As a result of this method, the position coordinates of the field source is obtained and the values are collected into the vector $\underline{x}_Q = \begin{Bmatrix} \xi_{xQ} \\ \xi_{yQ} \end{Bmatrix}$.

7. SOURCE TRACKING

The CAI (Centralized Artificial Intelligence) can use the information coming from the microphones to localize in the best way the source. To perform such task, it is necessary that carriers communicate to each other, in particular their measurements need to be synchronized. This need takes place in the so called *information sharing*, that is referred to one-to-one exchanges of data between a swarm of agents, in this case between the carriers and the CAI. Sharing tactical information of the agents, such as mutual positions and velocities of the surrounding environment is useful for several purposes: the collective exploration of the region *R*, i.e. mission *attack the target*, could be increased in terms of performance because the swarm can reach the target choosing the best trajectory with the optimal velocities and, in case of a large population of carriers, avoiding each other during the motion through a possible internal avoidance system. Once the coordinates of the field source ($\underline{x}_Q$) have been identified, the control algorithm adjusts the velocities of carriers in order to achieve the optimal trajectory to reach the engaged target. For the present case it has been developed a *Linear Quadratic Regulator* (i.e. LQR) as controller of the migration dynamics. Task of this algorithm is to obtain a feedback control law, function of the time-dependent state of the system. In detail the control equation is the following:

$$\underline{u} = -\underline{\underline{K}}_{LQR}\left(\underline{x} - \underline{x}_{target}\right) \tag{31}$$

that minimizes the specific cost function:

$$\min J = \int_0^T \frac{1}{2}\left(\underline{\tilde{x}}^T Q \underline{\tilde{x}} + \underline{u}^T R \underline{u}\right) dt \tag{32}$$

Where $\underline{\tilde{x}} = \underline{x} - \underline{x}_{target}$, subjected to the dynamic and initial conditions:

$$\begin{gathered} \underline{\dot{x}} = \underline{\underline{A}}\underline{x} + \underline{\underline{B}}\underline{u} \\ \underline{x}(0) = \underline{x}_0 \end{gathered} \tag{33}$$

In the present example it is reported the case of a controlled migration of a population of three carriers to a target point. Initial conditions, such as positions and velocities, have been randomly chosen. This problem could be easily extended to a larger number of agents. Supposed that within a population of agents, three of them perform the role of *masters*, once this portion of the swarm identifies the source, they can share the location data to the other carriers, called *slaves*, and begin the *attack the target*. Results for both cases are reported in the pictures below:

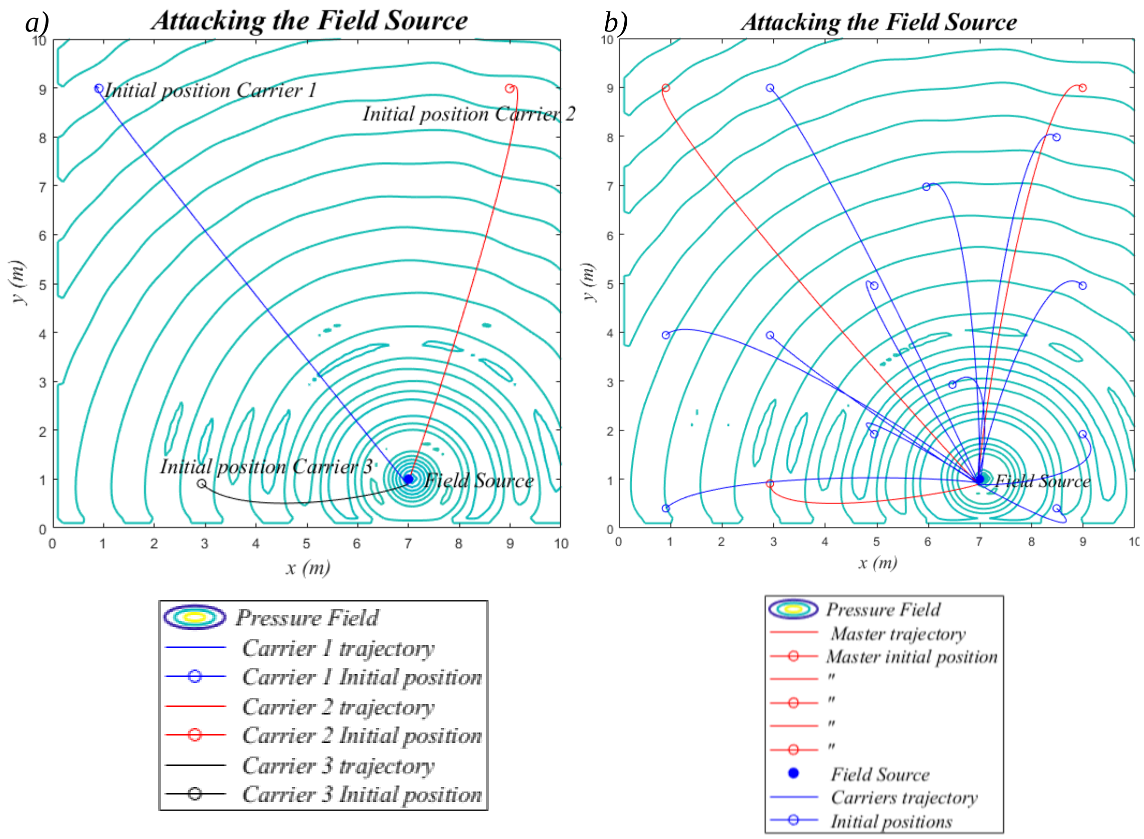

Figure 3 - Mission Attack the Target in case of a population of three carriers (a) and in the case of a larger population (b). Dots represent the initial position of the carriers, lines are the chosen trajectories.

In Figure (3a) the circles represent the initial position of the three carriers, while the lines represents the chosen trajectory to reach the target point. As shown in the picture, regardless of the initial motion conditions, through the feedback LQR control law, carriers are able to readjust their trajectory during the migration transient. In Figure (3b) same dynamic is reported for the case of the migration of a number of carriers equal to 15. In this last case, three carriers out of the entire population (the red ones in Figure (3b)), act as masters, and perform the role of identifying the coordinates of the source, and together with the remaining 12, called *slaves* (blue ones), perform the trapping mission.

In Figure (4) several shots of the swarm migration at difference time steps are reported. As it is shown, in the first two subplot (4.a and 4.b) all carriers, both masters and slaves, are mainly approaching the target position; in the last migration phase (4.c and 4.d) they settle their velocity, to eventually dispose themself around the field source. To clarify this behaviour, trend of velocity of one of the masters is reported in Figure (5): after an initial transient in which carriers are subjected to an acceleration, as the distance between them and the target decreases, the control action decreases with time as well (Equation 31). As a result, carriers slow down their motion untill they reach the target point $\underline{x}_T$ with velocity close to zero.

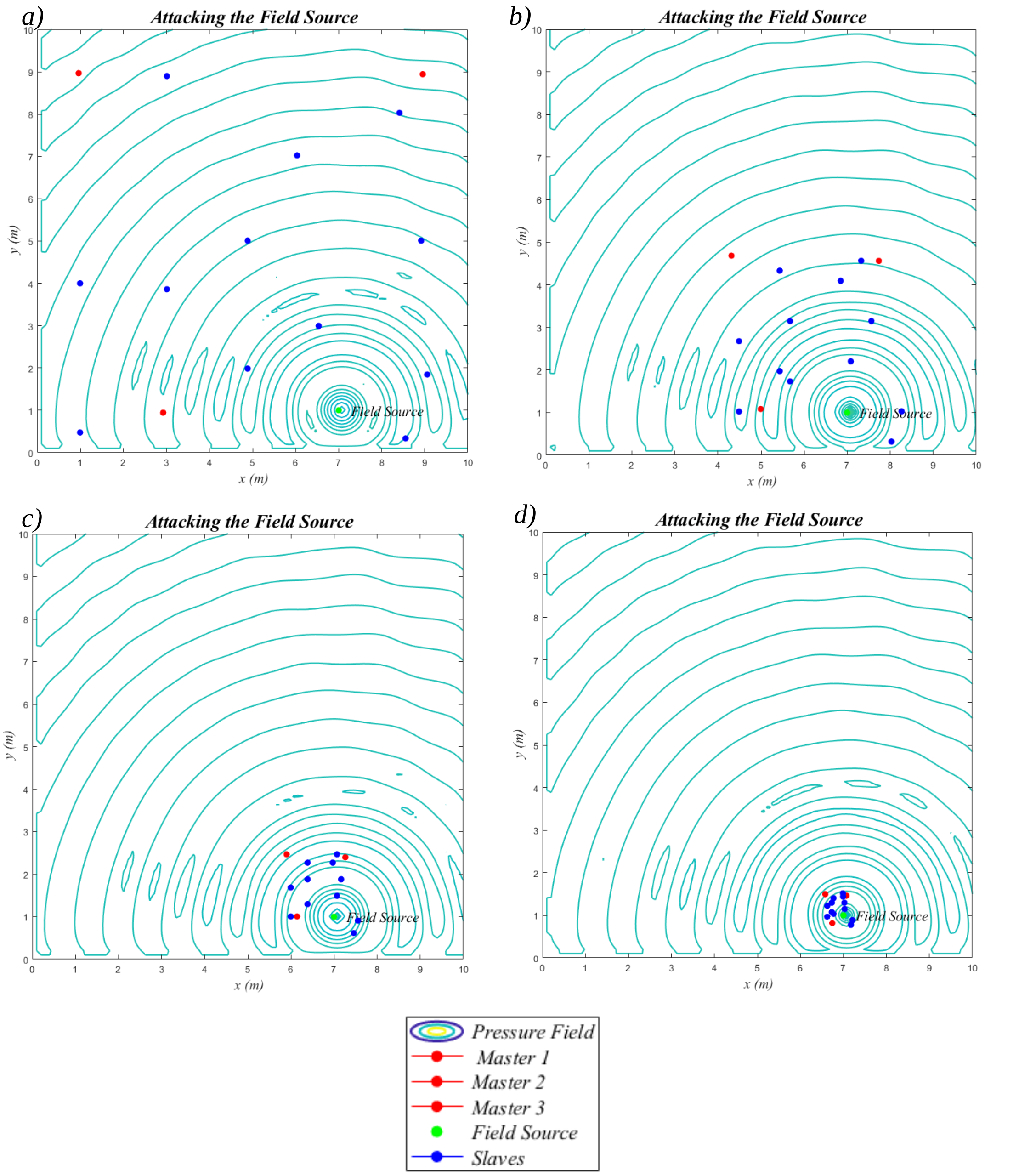

Figure 4 - Swarm migration at different time steps: a) t=0 sec; b) t=1.65 sec; c) t=2.83 sec; d=4.73 sec. Legend on the bottom of the picture.

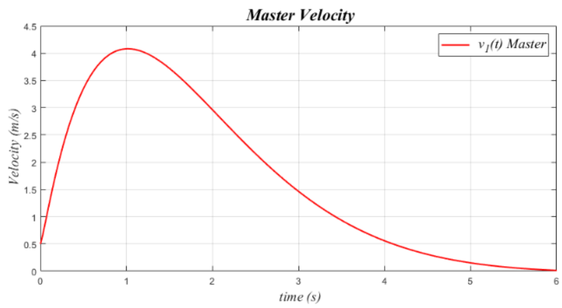

Figure 5 – velocity of one master during the swarm migration.

8. CONCLUSIONS

In this paper we present the outline of a new method for the swarm of sensors managing. The goal is to identify in the most effective way the characteristic of the acoustic field that is explored by the swarm.

The analysis leads to include in the model two different differential equations. One is related to the swarm dynamics, the second to the acoustic field.

The CAI includes three modules: RKF, SI, ST.

The first proposes to reduce the problem to a Kalman Filter that includes the two different models, while the third is the source tracking. These two control techniques are used combining their abilities. The module of Source Identification is always active.

The strategy is under investigation to be implemented on real cases in which the swarm is made by navigating small marine drones.

9. REFERENCES

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