

Overview of the phase and amplitude gradient estimator method for acoustic intensity

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ABSTRACT

This paper summarizes the theory, benefits, and example applications of a recently developed method for frequency-domain acoustic intensity estimation from multimicrophone probes. The phase and amplitude gradient estimator (PAGE) method can use the same hardware as the traditional method, but greatly improves bias error estimates for complex intensity by obtaining the gradient of the amplitude and phase separately. The theory described includes the bias error improvements, including phase unwrapping beyond the spatial Nyquist frequency. Applications to jet noise, outdoor sound source localization, and to improved broadband sound power estimates are described.

Keywords: Acoustic intensity, source localization, sound power I-INCE Classification of Subject Number: 72

1. INTRODUCTION

The acoustic intensity is an important quantity in sound field characterization and reconstruction, determination of radiated sound power, noise source identification, and determining building sound insulation. Obtaining the acoustic intensity involves the acquisition of collocated and synchronous pressure and particle velocity measurements. Because of the importance of intensity as an acoustic quantity, various measurement and signal processing methods have been developed and investigated since the first attempts to measure acoustic intensity in the 1930's [1], and numerous intensity-related standards now exist.

The so-called "p-p method" has traditionally used a pair of microphones located a known distance apart to estimate the intensity along the direction parallel to the line containing the microphones. [2, 3] A finite sum (average) is used to obtain the pressure estimate and a finite-difference (which leverages Euler's equation) the particle velocity estimate. In the frequency domain, the time-averaged intensity becomes a scaled, frequency-weighted quadspectrum between the two microphone signals. Multidimensional intensity estimates are obtained using orthogonal pairs of microphones or can leverage weighted least-squares estimates to require fewer intensity probe microphones. [26]

The traditional p-p method, first developed in the late 1970's[1], has proven enormously useful, leading to numerous applications. However, its principal drawback is that the measurement or calculation bandwidth is limited at high frequencies by finite sum/difference bias errors and at low frequencies by microphone mismatch (usually phase). Whiting et al. [4] built off the work of [5], [6] and [7] in showing that the traditional p-p method for intensity (hereafter referred to simply as the traditional method) for two microphones and a propagating plane wave reaches 5% error at kd < 0.55, where k is the acoustic wavenumber and d is the distance between microphones. This means the probe microphone spacing can be no more than $1/11^{\text{th}}$ the acoustic wavelength. Moreover, improving the intensity bandwidth at low frequencies by increasing the microphone separation distance, such that the acoustic phase difference between microphones becomes much larger than the instrumentation phase mismatch, reduces the high-frequency limits further.

The recently developed phase and amplitude gradient estimation (PAGE) method for acoustic intensity [8] improves upon the calculation bandwidth for multimicrophone probes. The PAGE method, instead of treating the real and imaginary parts of the complex pressures at the microphones like the traditional method, instead performs calculations on the magnitude and phase as the name suggests. The method has been studied in some detail analytically and has resulted the development of related signal processing methods for reactive intensity and specific acoustic impedance [4, 10], directional pressure estimation [22], and enhanced array performance in beamforming [11]. The PAGE method for intensity has been experimentally applied to loudspeaker sound field characterization [12, 13] and in rocket and jet aeroacoustic source characterization [14-17]. Another recent example has been the use of PAGE method in the real-time localization of unmanned aerial vehicles [18, 19].

This paper explains the PAGE method theory and its advantages compared to the traditional method, with validation measurements, as well as some limitations. Finally, select recent and current applications are presented to show the promise of the PAGE method in extending the utility of acoustic intensity measurements.

2. THEORY, BENEFITS, AND VALIDATION

2.1 Theory summary

The phase and amplitude gradient estimator (PAGE) method [8] involves the estimation of the least-squares gradient of the pressure phase across microphones on a probe to calculate the frequency-domain active intensity. The PAGE method estimates the gradient of phase and amplitude of complex pressure obtained from the Fourier transform of the data recorded by a probe of microphones, and expressed as a complex quantity at position r as a magnitude and phase:

$$p(\mathbf{r}) = P(\mathbf{r})e^{-j\phi(\mathbf{r})},\tag{1}$$

where the bold notation indicates a vector quantity. The position of the microphones in the array are represented in a matrix with the unique pairwise separation vectors in a row layout according to

$$X = [r_2 - r_1, \dots, r_N - r_{N-1}]^T.$$
 (2)

For a given frequency, the vector of pairwise phase differences is given by the pairwise transfer function as

$$\Delta \boldsymbol{\phi} = -[\arg\{H_{1,2}\}, \dots, \arg\{H_{N-1,N}\}]^T$$
(3)

Then, the time-average active intensity can be written as

$$\boldsymbol{I} = \frac{1}{2} Re\{\boldsymbol{p}\boldsymbol{u}^*\} = \frac{1}{2\rho_o \omega} P^2 \boldsymbol{\nabla} \boldsymbol{\phi} = \frac{1}{\rho_o \omega} \overline{P^2} \boldsymbol{\nabla} \boldsymbol{\phi}, \tag{4}$$

where * means complex conjugate, ρ_0 is the ambient density, and $\overline{P^2}$ is the mean-square pressure at frequency, ω . This pressure is found by locating a microphone at the center or, alternatively, by estimating the least-squares pressure magnitudes across the probe. The least-squares phase gradient is calculated from N microphones probe with $r_{1...N}$ position as:

$$\nabla \boldsymbol{\phi} \approx (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \Delta \boldsymbol{\phi}.$$
 (5)

2.2 Bias error reduction

It was shown by Whiting et al. [4] that the PAGE method dramatically reduces the active intensity bias errors up to the probe's spatial Nyquist frequency. For a propagating plane-wave, the errors are zero. Figure 1 below shows some results from a monopole source and a two-microphone probe, where r and d are defined in the figure. The solid diagonal line indicates where the intensity level error drops below 5% and the vertical dotted line for both methods represents the spatial Nyquist frequency ($kd = \pi$). For large kr, the results in Fig. 1 revert to the plane-wave case. In the very near field, the bias errors for the PAGE method are slightly greater than those of the traditional method, but do not worsen with frequency and can be extended beyond the spatial Nyquist frequency provided that phase unwrapping is an option.



Figure 1. Top: two-microphone probe configuration separated by d and located distance r from the source. Level bias errors for traditional p-p intensity and PAGE with phase unwrapping.

2.3 Phase unwrapping

The bias error reduction obtained with the PAGE method for an extended frequency band above the spatial Nyquist frequency of the probe is made possible by phase unwrapping. Use of the pressure magnitude and phase, rather than the real and imaginary parts makes possible the determination of the absolute phase gradient (rather than the 2π -limited gradient), which is required for accurate intensity measurements. The unwrapping process could be performed by the "unwrap.m" in MATLAB®, which adds $2n\pi$ to phases corresponding to the nth frequency interval for which there is a > π phase jump. Generally, the unwrapping works better for signals with smooth variation of $\nabla \phi$ as a function of frequency, which means that the phase gradient should not show abrupt variation over the frequency range of interest. For signal pairs with reduced coherence and signal-to-noise ratio, a new phase unwrapping process has been developed to improve the application of the PAGE method in the presence of noise. This method is explained by Cook et al. [20] and uses the coherence between the microphone pair to improve the unwrapping method. Phase unwrapping and other considerations for narrowband signals have been explored by Succo et al. [13]

2.4 Validation

A number of experiments have been performed to validate the PAGE method. First, demonstration of phase unwrapping was carried out [21] using an anechoic planewave tube and broadband propagating field, for which PAGE bias errors should be effectively zero. Figure 2a shows the plane wave setup, with microphones spaced as wide as 90 cm apart, and Fig. 2b shows the large bias errors in intensity level, relative to the sound pressure level. The phase unwrapping effect is shown in Figure 2c, where dashed lines are the wrapped data. This unwrapped phase is used to calculate the PAGE intensity levels, shown in Fig. 2d. As theoretically predicted, with phase unwrapping there are no bias errors up to the plane-wave tube cutoff frequency of about 2 kHz. Other bias-error reduction experiments for two and three-microphone 1D probes are reported on in [10].



Figure 2. (a) Plane-wave tube setup with microphones at different spacings. (b) Intensity and sound pressure levels calculated with the traditional method. (c) Wrapped and unwrapped phase for the different microphone spacings. (d) PAGE-calculated intensity levels.

Probe validation for 2D and 3D designs was carried out by Rose et al. [9], including a 2D equilateral triangle probe with a microphone in each vertex and an additional one in the center (see Fig. 3). Bias error reductions in sound intensity level magnitude and direction for the PAGE method were significant for higher frequencies. Figure 3 shows an example for a broadband loudspeaker experiment in an anechoic chamber, where the intensity probe was rotated 360 degrees to show errors in intensity magnitude and direction. The PAGE method greatly outperforms the traditional intensity method for both magnitude and direction. Note also that the results were compared with another probe configuration which had a smaller microphone spacing. The purpose of this comparison was to show that, unlike the traditional method, using a larger microphone spacing increases probe bandwidth at the low and high frequencies. A larger spacing reduces scattering and high-frequency errors while maintaining high-frequency performance because of the phase unwrapping. The larger spacing also improves lowfrequency intensity calculations because microphone pair phase mismatch becomes relatively smaller. [9] Overall, the PAGE method lowers intensity magnitude and direction errors, thus increasing probe bandwidth.



Figure 3. Top: 2D equilateral triangle probe with ½" diameter microphones and a 2" microphone spacing. Bottom: Comparison between traditional and PAGE methods for intensity level and direction errors.

2.5 Limitations

Some known limitations of the PAGE method exist. First, although the reactive intensity bias errors are also reduced for radiated fields [10, 4], the rapid physical phase changes in a highly reactive field do pose some constraints on phase unwrapping. Lawrence *et al.* [22] investigated application of the PAGE method and other higher-order intensity methods for reactive fields. Second, the presence of noise – whether incoherent with the source of interest, or coherent – does have some impact on the PAGE calculations. Cook *et al.* [26] have extended the work of Whiting *et al.* [4] for the finite signal-to-noise ratio case.

3. EXAMPLE APPLICATIONS

Three examples of the PAGE method that showcase its potential for improving traditional frequency-dependent source and field characterization are now described: characterization of high-frequency, laboratory-scale jet noise, localization of an outdoor, low-frequency sound source, and broadband sound power measurements.

3.1 Laboratory jet noise

Before the introduction of the PAGE method, broadband vector-based characterization of the noise from small-scale laboratory-scale jets was limited by probe bandwidth constraints. The PAGE method provides the opportunity to perform an unprecedented direct near-field characterization [17,21] of a small supersonic jet, and ultimately, to compare to full-scale behavior. [15]

The jet is a broadband and frequency-dependent source that radiates in all directions, with a principal radiation direction that is also frequency dependent. Thus, the PAGE-based experiment enables examination of these frequency-dependent characteristics. The experiment acquisition setting is explained in detail by Gee et al. [21], but the characterization was carried out using two-dimensional measurements with four-microphone, equilateral-triangle probes, with a microphone spacing of 25.4 mm (1.27 times the nozzle-diameter (Dj) of the jet). A photograph of a probe is shown in Fig. 4a, along with four example intensity measurement locations (I-IV) with respect to the jet shear layer. These near-field locations were selected because the 30° and 35° angles formed by II/IV and I/III, respectively, match the overall sound pressure level (OASPL) peak directivity range.

Additional representative results are displayed in Fig. 4 for the sample locations. Figure 4b shows successful phase unwrapping at location IV out to a frequency of at least 40 kHz, making PAGE-based intensity calculations feasible. Figure 4c shows a comparison between the sound pressure level (L_p) and the sound intensity level (L_I) calculated with both the traditional and PAGE methods. The failure of L_I^{TRAD} to match L_p for a propagating wave field is an indication of the significant bias errors of the traditional method above ~2.5 kHz. Figure 4d shows the intensity angle at all four locations in Fig. 4a. The intensity angle smoothly varies with frequency and location, but within the peak-frequency regions of the spectra in Fig. 4c, they mostly fall within $30 - 40^{\circ}$.

Figure 5 shows PAGE-calculated intensity vector maps around the jet for two frequencies: 2 kHz, which is below the peak frequency region of the jet, and 40 kHz, which would be impossible to calculate with a multimicrophone probe and the traditional method. However, with the PAGE method and phase unwrapping, the intensity source and radiation directions are clearly revealed. A comparison of the two maps shows how the source location, extent, and radiated directivity changes with frequency. This laboratory-scale jet example has shown how the PAGE method can increase the probe

bandwidth by about 30 times and how the intensity method may be used for broadband sources well into the ultrasonic regime.



Figure 4. a) Laboratory jet intensity measurements, which four specific locations highlighted. (b) Phase Unwrapping example at location IV. (c) Traditional and PAGE intensity level spectra, compared with the sound pressure level spectrum. (d) PAGEpredicted intensity angles for the four locations in (a).



Figure 5. PAGE-calculated vector intensity for the lab-scale jet at (left) 2 kHz and (right) 40 kHz.

3.2 Outdoor source localization

An additional, ongoing research application of the PAGE method for acoustic intensity has been outdoor localization of low-frequency sound sources using probe setup with wide microphone spacing. Multiple tests have been performed each with a setup containing a triangle "probe" with a microphone in the center. The radius of the probe was 10 ft (~3 m) with triangle side lengths of 17.3 feet (5.3 meters) with ± 1 -inch uncertainty based on human error. The $\frac{1}{2}$ " microphones were each housed in a BYU-developed windscreen housing and ground-plate setup for measurements in adverse weather conditions. A subwoofer was positioned at varying distances from one edge of the triangle. A SOUNDBOKS 2 subwoofer was set at a specific frequency and recorded for 10 minutes. Figure 6 shows this setup for one specific recording session.

Sound source localization results are shown in Fig. 6 as a function of frequency, for both the traditional and PAGE methods. Above the \sim 35 Hz roll off frequency of the sound source, both the PAGE and traditional methods localize the source for approximately 30-40 Hz, before the traditional method diverges from the actual intensity angle. On the other hand, the PAGE method continues to localize the source up to the maximum analysis frequency of 1 kHz. This example shows the potential for the PAGE method to be used down into the infrasound range – with widely spaced microphones – while able to achieve vector characterizations into the audible range as well.



Figure 6. Outdoor source localization experimental setup and results.

3.3 Sound power

The PAGE method has been also used to improve upon the standardized method (ANSI \$12.19-1992) for using intensity to find the radiated sound power of a broadband source. An example of the capabilities of the PAGE method in extending the reliable bandwidth of sound power calculations is shown for a household blender. As a benchmark, the sound power spectrum for the blender was obtained according to the ISO 3741:1999 in a reverberation chamber. Using the same blender, acoustic intensity was measured in an anechoic chamber with a reflecting plane to estimate the sound power according to engineering standard ANSI S12.12-1992. The center of the blender was 0.5 meters above the reflecting plane. The top scanning surface of the one-meter cube is illustrated in Figure 7a below. The blender's sound power was estimated using intensity obtained from both the traditional (TRAD) and PAGE methods. The intensity-based sound power with the TRAD and PAGE processing are compared to the reverberation chamber benchmark in Figure 7b. While both the TRAD and PAGE sound power estimates match the reverberation chamber benchmark within 2 dB over the 300-1000 Hz band, the PAGE estimate continues to follow the benchmark up to and above the spatial Nyquist. The agreement above the spatial Nyquist requires successful phase unwrapping. Thus, the PAGE method extends the bandwidth over which accurate sound power estimations can be obtained.



Figure 7. a) Photo of the blender on a reflecting plane in the anechoic chamber with a superimposed schematic of top plane of scanning surface showing an intensity vector and normal vector. b) One-third octave band sound power comparing the reverberation chamber measurements (blue) to the TRAD (black) and PAGE (red) intensity-based sound power. The spatial Nyquist frequency is shown with the dashed vertical line.

4. CONCLUSION

This paper has summarized the PAGE method and its benefits in reducing bias errors and shown example applications. Although limitations exist, for radiated sound fields from sources with some broadband component, the PAGE method can greatly extend existing probe hardware's bandwidth through phase unwrapping.

The PAGE method has been applied to acoustic fields where the frequency range of interest was mainly in the audible range, nevertheless it could be applied to infrasound intensity measurements as well. Some sources to be analyzed in future work will include acoustic intensity calculation of outdoor sources in the infrasound regime in outdoors where wind noise, partial coherence, and microphone phase mismatch as the microphone cutoff frequency is approached will be a further challenge for the application of the PAGE method.

5. ACKNOWLEDGEMENTS

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