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Analytical approach for the analysis of multilayer rubber bearings based on fulfilment of the equations of internal equilibrium

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ABSTRACT

In this work, a new analytical approach for the analysis of multilayer rubber bearings is proposed. The method is based strictly on fulfilment of the equations of internal equilibrium that are almost independent of the shape of the boundary. The approach depends only on static values such as the area and moments of inertia of the contour shape. Previous studies have also considered the rigorous fulfilment of the equations of internal equilibrium, both in statics and in dynamics, as well as their application in the industry. Although these approaches report solutions that also satisfy the equilibrium equations at the boundary, they have only been applied to circular and annular cross-section shapes, as opposed to the greater generality of the solutions presented in this work. The analytical solutions obtained here are also mathematically very simple compared with those of previous studies. This approach can be used in the context of configurations designed to isolate vibrations and to optimize, in a simple way, the elastics parameters of multilayer rubber bearings. Results obtained using the proposed approach are compared with those obtained through the finite element method.

Keywords: Vibration, Materials, Rubber bearings
I-INCE Classification of Subject Number: 42

1. INTRODUCTION

In this work a new analytical approach for the analysis of multilayer rubber bearings which are widely used in civil, mechanical, and automotive engineering applications, is applied to the study of the transmissibility of forces and displacements in a multilayer structure.

In 1954, Freyssinet [1] proposed the idea of reinforcing rubber blocks with thin steel plates. These rubber bearings combine the vertical stiffness of a rubber pad and the horizontal flexibility of rubber reinforced by thin steel plates perpendicular to the vertical load.

Evaluation of the horizontal, vertical, and bending stiffnesses is very important to predict the dynamic response and to design efficient applications of multilayer elastomeric bearings. Research on the proper design of these vibration-isolation systems for buildings, bridges, nuclear facilities, and other kind of structures has also been reported more recently. They have included theoretical, numerical and experimental studies [2, 3, 4], although they used different approaches and assumptions [5]. Among them are the works of Gent and Lindley [6] and Gent and Meinecke [7], in which they assumed use of an incompressible material, and the studies by Chalhoub and Kelly [8, 9, 10], in which the material was treated as compressible. Although these approaches have presented rigorous solutions to the vibration problem, they have limitations [11]. In all of these works, two types of assumptions were made: kinematic assumptions about deformation and assumptions about the state of stress, which led to the approximate fulfillment of the internal equilibrium equations and rigorous fulfillment of the equilibrium equations at the boundary. These kinds of solutions depend on the shape of the boundary and present some mathematical complexity. Using a different approach, simplified formulae have also been presented to facilitate the design of elastomeric bearings [12]. Research on multilayer bearings continues, including theoretical and applied studies [13, 14, 15].

The method used in this paper [16] is based strictly on fulfilment of the equations of internal equilibrium that are almost independent of the shape of the boundary. The approach depends only on static values such as the area and moments of inertia of the contour shape. Previous studies have also considered the rigorous fulfilment of the equations of internal equilibrium, both in statics and in dynamics, as well as their application in the industry. Although these approaches report solutions that also satisfy the equilibrium equations at the boundary, they have only been applied to circular and annular cross-section shapes, as opposed to the greater generality of the solutions presented in this work. The analytical solutions obtained here are also mathematically very simple compared with those of previous studies. This approach can be used in the context of configurations designed to isolate vibrations and to optimize, in a simple way, the elastics parameters of multilayer rubber bearings

The paper has been organized as follows. In the introductory section both the problem and the solution approach are presented. In the second section a harmonic analysis of a problem consisting of two floating sheets and two floating slabs is presented. The cases of transmissibility of forces and displacements are studied. Section 3 presents a numerical example and the conclusions are presented in the last section.

2. HARMONIC ANALYSIS

Consider the structure showed in figure 1, where two rectangular Cartesian coordinate system are defined relative to an origin located at Q_1y Q_2 , respectively . G_1 y G_2 are the mass center of the plates 1 and 2, O_1 and O_2 is the center points of the lower face of the plate (upper face of the elastics layers). Thus u(x, y,z, t), v(x, y,z, t), and w(x, y,z, t) are the displacements of the points of the elastic layer as a function of the coordinates x, y, z and time t.

Note that $h = m/\rho_p A$ is the thickness of each plate, where m and ρ_p are the mass and density, respectively. The elastic properties of the elastic layer materials are described by its Poisson's ratio (ν), Young's modulus (E), bulk modulus (κ), shear modulus (G), Lame's first parameter (λ), and P-wave modulus (M).

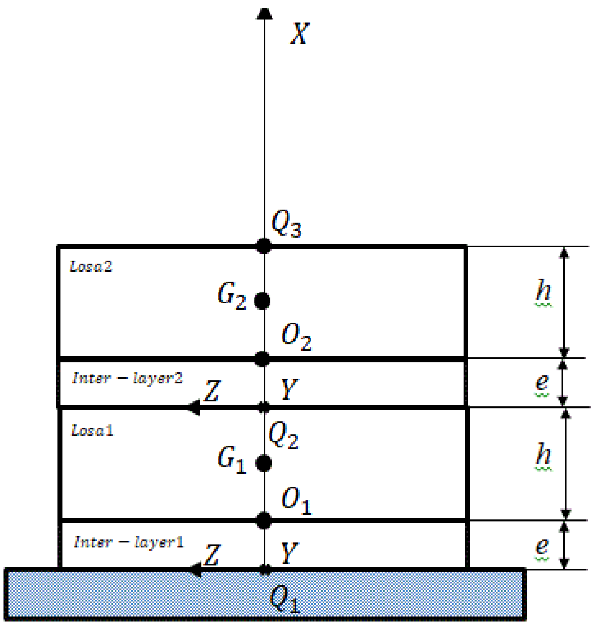

Figure 1. Geometry of the problem

Usually, there are two types of problems: the transmissibility of forces and the transmissibility of displacements.

The following assumptions are made:

1. No slip is allowed at the bonding surface between the plates and the elastic layers or between the elastic layer and the rigid foundation.
2. The plates stiffness is much greater than that of the elastic layers, so the plates can be considered ~~a~~ two rigid bodies.
3. The thickness of the elastic layers are much lesser than their lateral dimensions.
4. The displacement gradients in the elastic layers remain sufficiently small throughout the subsequent deformations, so it is permissible to apply the classical linear theory of elasticity.
5. For harmonic analyses, the damping of the plates can be neglected, and the damping of the elastic layers can be determined from the imaginary part of their elastic parameters.

Since the plates are assumed to be two rigid bodies, we can use the fundamental equations for the motion of rigid bodies in three dimensions [16].

The equations that describe the transmissibility of forces are

$$F_{X_{2,Q_2}} + F_{X_{1,O_1}} = m\ddot{u}_{G_1} \tag{1}$$

$$F_{Y_{2,Q_2}} + F_{Y_{1,O_1}} = m\ddot{v}_{G_1} \tag{2}$$

$$F_{Z_{2,Q_2}} + F_{Z_{1,O_1}} = m\ddot{w}_{G_1} \tag{3}$$

$$M_{X_{2,Q_2 \to G_1}} + M_{X_{1,O_1 \to G_1}} = I_{XG_1}\ddot{\theta}_{XG_1} \tag{4}$$

$$M_{Y_{2,Q_2\to G_1}} + M_{Y_{1,O_1\to G_1}} = I_{YG_1}\ddot{\theta}_{YG_1} \tag{5}$$
$$M_{Z_{2,Q_2\to G_1}} + M_{Z_{1,O_1\to G_1}} = I_{ZG_1}\ddot{\theta}_{ZG_1} \tag{6}$$
$$F_{X,ext} + F_{X_{2,O_2}} = m\ddot{u}_{G_2} \tag{7}$$
$$F_{Y,ext} + F_{Y_{2,O_2}} = m\ddot{v}_{G_2} \tag{8}$$
$$F_{Z,ext} + F_{Z_{2,O_2}} = m\ddot{w}_{G_2} \tag{9}$$
$$M_{X,ext\to G_2} + M_{X_{2,O_2\to G_2}} = I_{XG_2}\ddot{\theta}_{XG_2} \tag{10}$$
$$M_{Y,ext\to G_2} + M_{Y_{2,O_2\to G_2}} = I_{YG_2}\ddot{\theta}_{YG_2} \tag{11}$$
$$M_{Z,ext\to G_2} + M_{Z_{2,O_2\to G_2}} = I_{ZG_2}\ddot{\theta}_{ZG_2} \tag{12}$$

where the principal centroidal moments of inertia of the plate are

$$I_{XG_j} = m\frac{A}{6} \tag{13}$$
$$I_{YG_j} = I_{ZG_j} = \frac{m}{12}(A + h^2)$$

$F_{Z_{2,Q_2}}$ *is* the component Z of the forces transmitted by sheet 2 through the surface where is Q_2; $M_{Z_{1,O_1\to G_1}}$ is the component Z of the moment resulting from the forces transmitted by sheet 1 across the surface where is $O_1$and reduced to G_1 and $M_{X,ext\to G_2}$ is the component X of the moment resulting from the forces transmitted from the outside and reduced to G_2.

Equations 1 to 6 correspond to the dynamic equilibrium of slab 1 as a rigid solid. It should be noted that in the case of transmissibility of displacements, the terms due to the contribution from the outside disappear in equations from 7 to 12. ($F_{X,ext}, F_{Y,ext}, F_{Z,ext}, M_{X,ext\to G_2}, M_{Y,ext\to G_2}$ y $M_{Z,ext\to G_2}$)

To determine the linear and angular displacements of the mass center, we use the following linear approximation [16] between two points P and Q of a plate

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_P \approx \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_Q + \vec{\theta} \times \overrightarrow{QP} \tag{14}$$

is taken as reference for the calculation of the rotations of the plate j at point O_j, which is the same as that of the point Q_{j+1}.

We consider that the movements for each plate j, are given by the following equations

$$\begin{aligned} u_j(x,y,z,t) &= e^{i\omega t} f_{uj}(x) g_{uj}(y,z) \\ v_j(x,z,t) &= e^{i\omega t} f_{vj}(x) g_{vj}(z) \\ w_j(x,y,t) &= e^{i\omega t} f_{wj}(x) g_{wj}(y) \end{aligned} \tag{15}$$

Next, the general methodology explained in [18-19], will be applied to the two basic problems of transmissibility (displacements and forces). The boundary conditions given by the following equations for each plate j can be considered:

$$u_j(x=0,y,z,t) = \mathrm{e}^{\mathrm{i}\omega\mathrm{t}}\left(z\,\theta_{\mathrm{YQ}_j} - y\,\theta_{\mathrm{ZQ}_j} + u_{\mathrm{Q}_j}\right)$$
$$v_j(x=0,y,z,t) = \mathrm{e}^{\mathrm{i}\omega\mathrm{t}}\left(-z\,\theta_{\mathrm{XQ}_j} + v_{\mathrm{Q}_j}\right)$$

$$w_j(x=0,y,z,t) = \mathrm{e}^{\mathrm{i}\omega t}\left(y\,\theta_{\mathrm{XQ}_j} + w_{\mathrm{Q}_j}\right) \tag{16}$$

(In the case of force transmissibility, the six movements of the plate j = 1 are null).

Substituting equations 15 into the Lamé-Navier equations in Cartesian coordinates [17], we obtain the equations of the displacement in each plate j as

$$u_j(x,y,z,t) = \mathrm{e}^{\mathrm{i}\omega t}\Big(\mathrm{Sin}(k_p x)\left(d_{-5+6j} + y d_{-4+6j} + z d_{-3+6j}\right) + \mathrm{Cos}(k_p x)\left(z\,\theta_{\mathrm{YQ}_j} - y\,\theta_{\mathrm{ZQ}_j} + u_{\mathrm{Q}_j}\right)\Big) \tag{17}$$

$$v_j(x,y,z,t) = \mathrm{e}^{\mathrm{i}\omega t}\frac{\left(-\mathrm{Cos}(k_p x) + \mathrm{Cos}(k_s x)\right) d_{-4+6j} - \mathrm{Sin}(k_p x)\,\theta_{\mathrm{ZQ}_j}}{k_p} + \mathrm{e}^{\mathrm{i}\omega t}\left(\mathrm{Sin}(k_s x)(z d_{6j} + d_{-2+6j})\mathrm{Cos}(k_s x)\left(-z\,\theta_{\mathrm{XQ}_j} + v_{\mathrm{Q}_j}\right)\right) \tag{18}$$

$$w_j(x,y,z,t) = \mathrm{e}^{\mathrm{i}\omega t}\frac{\left(-\mathrm{Cos}(k_p x) + \mathrm{Cos}(k_s x)\right) d_{-3+6j} + \mathrm{Sin}(k_p x)\,\theta_{\mathrm{YQ}_j}}{k_p} + {} + \mathrm{e}^{\mathrm{i}\omega t}\left(\mathrm{Sin}(k_s x)\left(-y d_{6j} + d_{-1+6j}\right) + \mathrm{Cos}(k_s x)\left(y\,\theta_{\mathrm{XQ}_j} + w_{\mathrm{Q}_j}\right)\right) \tag{19}$$

To express the 12 equations of dynamic equilibrium (1–12) as a system of 12 linear algebraic equations with 12 unknowns d_i (i = 1, ... 12), we must:

1) Bear in mind that, in the problem of transmissibility of movements, $u_{\mathrm{Q}_1}, v_{\mathrm{Q}_1}, w_{\mathrm{Q}_1}, \theta_{\mathrm{XQ}_1}, \theta_{\mathrm{YQ}_1}\ y\ \theta_{\mathrm{ZQ}_1}$ are known values (the movements to be transmitted and that they want to be reduced), while in the problem of transmissibility of forces these values are zero to be able to calculate the reactions transmitted to point Q_1, which will be expressed in terms of the 12 unknowns di (i=1,…,12). Thus, with the stress field in layer 1:

$$(R_{FX}, R_{FY}, R_{FZ}) = \iint_A \left(-\sigma_x, -\tau_{xy}, -\tau_{xz}\right)_{x=x\ del\ punto\ \mathrm{Q}_1 de\ lámina\ 1} dA$$
$$(R_{MX}, R_{MY}, R_{MZ}) = \iint_A \left(z\tau_{xy} - y\tau_{xz}, -z\sigma_x, y\sigma_x\right)_{x=x\ del\ punto\ \mathrm{Q}_1 de\ lámina\ 1} dA \tag{20}$$

2) The rest of the forces exerted on the two plates will be expressed according to the 12 unknowns d_i (i=1,…,12), such that:
   a) Plate 1:
      i) With the stress field of layer 1:

$$(F_{X_{1,O_1}}, F_{Y_{1,O_1}}, F_{Y_{1,O_1}}) = \iint_A \left(-\sigma_x, -\tau_{xy}, -\tau_{xz}\right)_{x=x\ del\ punto\ \mathrm{O}_1} dA$$
$$(M_{X_{1,O_1\to G_1}}, M_{Y_{1,O_1\to G_1}}, M_{Z_{1,O_1\to G_1}}) = \iint_A \left(z\tau_{xy} - y\tau_{xz}, -\frac{h}{2}\tau_{xz} - z\sigma_x, \frac{h}{2}\tau_{xy} + y\sigma_x\right)_{x=x\ del\ punto\ \mathrm{O}_1} dA \tag{21}$$

      ii) With the stress field of layer 2:

$$F_{X_{2,Q_2}}, F_{Y_{2,Q_2}}, F_{Y_{2,Q_2}}) = \iint_A \left(-\sigma_x, -\tau_{xy}, -\tau_{xz}\right)_{x=x\ del\ punto\ \mathrm{Q}_2} dA$$

$$(M_{X_{2,Q_2\to G_1}}, M_{Y_{2,Q_2\to G_1}}, M_{Z_{2,Q_2\to G_1}}) = \iint_A \left(z\tau_{xy} - y\tau_{xz}, \frac{h}{2}\tau_{xz} - z\sigma_x, -\frac{h}{2}\tau_{xy} + y\sigma_x\right)_{x=x\ del\ punto\ \mathrm{Q}_2} dA \tag{22}$$

b) Plate 2:

i) With the stress field of layer 2:

$$(F_{X_{2,O_2}}, F_{Y_{2,O_2}}, F_{Y_{2,O_2}}) = \iint_A \left(-\sigma_x, -\tau_{xy}, -\tau_{xz}\right)_{x=x\ del\ punto\ \mathrm{O}_2} dA$$

$$(M_{X_{2,O_2\to G_2}}, M_{Y_{2,O_2\to G_2}}, M_{Z_{2,O_2\to G_2}}) = \iint_A \left(z\tau_{xy} - y\tau_{xz}, -\frac{h}{2}\tau_{xz} - z\sigma_x, \frac{h}{2}\tau_{xy} + y\sigma_x\right)_{x=x\ del\ punto\ \mathrm{O}_2} dA \tag{23}$$

3) In addition, when performing the previous integrals, so that only 12 unknowns remain, it is necessary to use these equations, where the term $e^{i\omega t}$ has been omitted:

- Between points O_j and Q_j of each layer j, from the field equations $u_j(x,y,z,t), v_j(x,y,z,t), w_j(x,y,z,t)$ at $x = e, y = 0, z = 0$ the following equations are obtained:

$$\begin{aligned}
u_{\mathrm{O}_j} &= \mathrm{Sin}(k_p e)\, d_{-5+6j} + \mathrm{Cos}(k_p e)\, u_{\mathrm{Q}_j} \\
v_{\mathrm{O}_j} &= \mathrm{Sin}(k_s e) d_{-2+6j} - \mathrm{Cos}(k_p e)\frac{d_{-4+6j}}{k_p} - \mathrm{Sin}(k_p e)\frac{\theta_{\mathrm{ZQ}_j}}{k_p} \\
&\quad + \mathrm{Cos}(k_s e)\left(\frac{d_{-4+6j}}{k_p} + v_{\mathrm{Q}_j}\right) \\
w_{\mathrm{O}_j} &= \mathrm{Sin}(k_s e) d_{-1+6j} + \mathrm{Cos}(k_p e)\left(-\frac{d_{-3+6j}}{k_p}\right) + \mathrm{Sin}(k_p e)\left(\frac{\theta_{\mathrm{YQ}_j}}{k_p}\right) \\
&\quad + \mathrm{Cos}(k_s e)\left(\frac{d_{-3+6j}}{k_p} + w_{\mathrm{Q}_j}\right)
\end{aligned} \tag{24}$$

- Between the mass center G_j of plate j and point Q_j of layer j, and using the kinematic equations for the rigid plate j, we get:

$$\begin{aligned}
u_{\mathrm{G}_j} &= \mathrm{Sin}(k_p e)\, d_{-5+6j} + \mathrm{Cos}(k_p e)\, u_{\mathrm{Q}_j} \\
v_{\mathrm{G}_j} &= \mathrm{Sin}(k_s e) d_{-2+6j} + \mathrm{Cos}(k_p e)\left(-\frac{d_{-4+6j}}{k_p} + \frac{h\,\theta_{\mathrm{ZQ}_j}}{2}\right) \\
&\quad + \mathrm{Sin}(k_p e)\left(-\frac{h d_{-4+6j}}{2} - \frac{\theta_{\mathrm{ZQ}_j}}{k_p}\right) + \mathrm{Cos}(k_s e)\left(\frac{d_{-4+6j}}{k_p} + v_{\mathrm{Q}_j}\right)
\end{aligned} \tag{25}$$

$$w_{\mathrm{G}_j} = \mathrm{Sin}(k_s e) d_{-1+6j} + \mathrm{Cos}(k_p e)\left(-\frac{d_{-3+6j}}{k_p} - \frac{h\,\theta_{\mathrm{YQ}_j}}{2}\right) + \mathrm{Sin}(k_p e)\left(-\frac{h d_{-3+6j}}{2} + \frac{\theta_{\mathrm{YQ}_j}}{k_p}\right) + \mathrm{Cos}(k_s e)\left(\frac{d_{-3+6j}}{k_p} + w_{\mathrm{Q}_j}\right)$$

Now, for the rotations we obtain:

$$\begin{aligned} \theta_{\mathrm{XG}_j} &= -\mathrm{Sin}(k_s e) d_{6j} + \mathrm{Cos}(k_s e)\theta_{\mathrm{XQ}_j} \\ \theta_{\mathrm{YG}_j} &= \mathrm{Sin}(k_p e) d_{-3+6j} + \mathrm{Cos}[k_p e]\theta_{\mathrm{YQ}_j} \\ \theta_{\mathrm{ZG}_j} &= -\mathrm{Sin}(k_p e) d_{-4+6j} + \mathrm{Cos}[k_p e]\theta_{\mathrm{ZQ}_j} \end{aligned} \tag{26}$$

- Between points Q_{j+1} y Q_j associated to each plate j, from the equations for the rigid plate j and the field equations $u_j(x,y,z,t), v_j(x,y,z,t), w_j(x,y,z,t)$ at $x = e, y = 0, z = 0$, the following equations are derived:
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$$u_{\mathrm{Q}_{j+1}} = \mathrm{Sin}(k_p e) d_{-5+6j} + \mathrm{Cos}(k_p e) u_{\mathrm{Q}_j}$$

$$v_{\mathrm{Q}_{j+1}} = \mathrm{Sin}(k_s e) d_{-2+6j} + \mathrm{Cos}(k_p e)\left(-\frac{d_{-4+6j}}{k_p} + h\,\theta_{\mathrm{ZQ}_j}\right) - \mathrm{Sin}(k_p e)\left(h\,d_{-4+6j} + \frac{\theta_{\mathrm{ZQ}_j}}{k_p}\right) + \mathrm{Cos}(k_s e)\left(\frac{d_{-4+6j}}{k_p} + v_{\mathrm{Q}_j}\right) \tag{27}$$

$$w_{\mathrm{Q}_{j+1}} = \mathrm{Sin}(k_s e) d_{-1+6j} - \mathrm{Cos}(k_p e)\left(\frac{d_{-3+6j}}{k_p} + h\,\theta_{\mathrm{YQ}_j}\right) + \mathrm{Sin}(k_p e)\left(-h\,d_{-3+6j} + \frac{\theta_{\mathrm{YQ}_j}}{k_p}\right) + \mathrm{Cos}(k_s e)\left(\frac{d_{-3+6j}}{k_p} + w_{\mathrm{Q}_j}\right)$$

$$\begin{aligned} \theta_{\mathrm{XQ}_{j+1}} &= -\mathrm{Sin}(k_s e) d_{6j} + \mathrm{Cos}(k_s e)\theta_{\mathrm{XQ}_j} \\ \theta_{\mathrm{YQ}_{j+1}} &= \mathrm{Sin}(k_p e) d_{-3+6j} + \mathrm{Cos}(k_p e)\theta_{\mathrm{YQ}_j} \\ \theta_{\mathrm{ZQ}_{j+1}} &= -\mathrm{Sin}(k_p e) d_{-4+6j} + \mathrm{Cos}(k_p e)\theta_{\mathrm{ZQ}_j} \end{aligned} \tag{28}$$

Once equations 1 to 12 have been expressed as a function of only 12 unknowns, di, for each frequency, we can now solve the system of linear equations and obtain their value.

a) For the problem of transmissibility of displacements: the first step is to apply equations 28 and 27 with j = 1 to obtain the six movements of the point Q_2. Then, these equations are applied again with j = 2 to determine the movements of point Q_3.
b) For the problem of transmissibility of forces, we try to apply equations 20 to determine the six components of the reaction.

3. NUMERICAL EXAMPLE

An application of the described methodology has been performed to test the theory presented above for the structure shown in figure 1 with two solid plates ($\mathrm{m} = 8\ \mathrm{kg}, \mathrm{h} = 0.0255\ \mathrm{m.}$ and $\mathrm{A} = 0.04\ \mathrm{m}^2$) and two elastic layers ($\rho = 20\frac{kg}{m^3}, e = 0.003\ m, M = 143951\ Pa, \nu = 0.45$).

3.1 Transmissibility of forces

Figure 2 shows the frequency-dependent force transmissibility function.

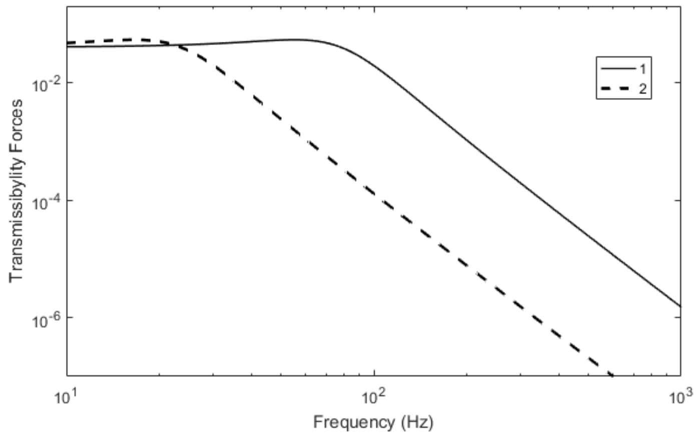

Figure 2. Transmissibility of forces 1. $R_{FX}/F_{X,ext}$, 2. $R_{FY}/F_{Y,ext}$ as a function of frequency.

3.2 Transmissibility of displacements

Figure 3 shows the frequency-dependent displacement transmissibility function for u, v and w.

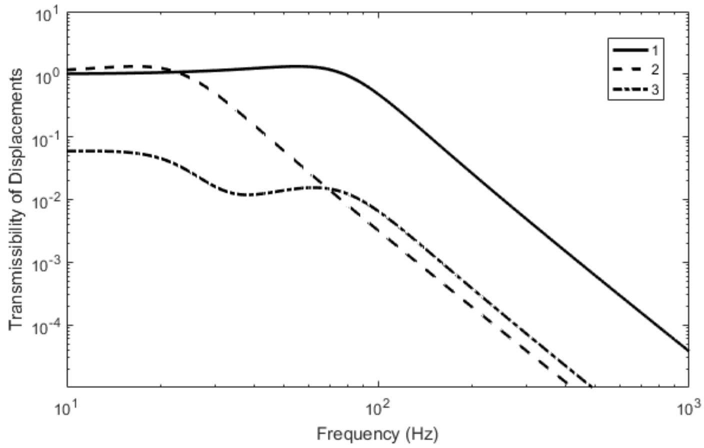

Figure 3. Transmissibility of displacements 1. u_{Q3}/u_{Q1} 2. v_{Q3}/v_{Q1}3. w_{Q3}/w_{Q1} as a function of frequency.

4. CONCLUSIONS

A new analytical approach for the analysis of multilayer rubber bearings has been proposed. The method is based strictly on fulfilment of the equations of internal equilibrium that are almost independent of the shape of the boundary. The approach depends only on static values such as the area and moments of inertia of the contour shape. This method has been applied to solve a simple problem of transmissibility of forces and displacements with only two elastic layers and two plates. However, it is possible to generalize the formulation for any number of layer/plate combinations.

5. REFERENCES

1. E. Freyssinet. *Dispositif de liaison elastique a un ou plusieurs degres de liberte* (translated as Elastic Device of Connection to One or More Degrees of Freedom). French patent no. 1.110.285 Class F06 d 25/5/54. Ministry of Industry and Commerce, France, 1954.

2. Z.-D. Xu, S.-A. Wang, C. Xu, *Experimental and numerical studyon long-span reticulate structure with multidimensional high-dampingearthquake isolation devices*, J. Sound Vib. 333 (2014) 3044–3057. https://doi.org/10.1016/j.jsv.2014.02.013.

3. K.N. Kalfas, S.A. Mitoulis, K. Katakalos, *Numerical study on the response of steel-laminated elastomeric bearings subjected to variable axial loadsand development of local tensile stresses*, Eng. Struct. 134 (2017) 346–357.https://doi.org/10.1016/j.engstruct.2016.12.015.

4. M. Ostberg, L. Kari, *Transverse, tilting and cross-coupling sti ffness of cylindrical* rubber isolators in the audible frequency range – The wave-guide solution, J. Sound Vib. 330 (2011) 3222–3244. https://doi.org/10.1016/j.jsv.2011.01.020.

5. J.M. Kelly, *Earthquake-Resistant Design with Rubber*, second ed., Springer, London, 1997. https://doi.org/10.1007/978-1-4471-0971-6.

6. A.N. Gent, P.B. Lindley, The compression of bonded rubber block, Proc. Inst. Mech. Eng. 173 (1959) 111–122. https://doi.org/10.1243/pime proc 1959 173 022 02.28

7. A.N. Gent, E.A. Meinecke, *Compression, bending and shearof bonded rubber blocks*, Polymer Eng. Sci. 10 (1970) 48–53.https://doi.org/10.1002/pen.760100110.

8. M.S. Chalhoub, J.M. Kelly, *Reduction of the Stiffness of Rubber Bearings due to Compressibility, Report No. UCB/SEMM-86/06*, Department of Civil Engineering, University of California, Berkeley, 1987.

9. M.S. Chalhoub, J.M. Kelly, Effect of bulk compressibility on the stiffness of cylindrical base isolation bearings, Int. J. Solids Struct. 26 (1990) 743–760. https://doi.org/10.1016/0020-7683(90)90004-f.

10. M.S. Chalhoub, J.M. Kelly, *Analysis of infinite-strip-shaped base isolator with elastomer bulk compression*, J. Eng. Mech. (ASCE) 117 (1991) 1791– 1805. https://doi.org/10.1061/(asce)0733-9399(1991)117:8(1791).

11. M.H.B.M. Shariff, *A general approach to axial deformation of bonded elastic mounts of various cross-sectional shapes*, Appl. Math. Model. 17 (1993) 430–436. https://doi.org/10.1016/0307-904x(93)90118-z.

12. H.-C. Tsai, W.-J. Pai, *Simplified stiffness formulae for elastic layers bonded between rigid plates*, Eng. Struct. 25 (2003) 1443–1454. https://doi.org/10.1016/s0141-0296(03)00044-0.

13. G. Barone, M. Di Paola, F. Lo Iacono, G. Navarra, *Viscoelastic bearings with fractional constitutive law for fractional tuned mass dampers*, J. Sound Vib. 344 (2015) 18–27. https://doi.org/10.1016/j.jsv.2015.01.017.

14. L. Wang, S. Li, S. Yao, D. Lv, P. Jia, *Study on the vertical stiffness of the spherical elastic layer bonded between rigid surfaces*, Arch. Appl. Mech. 87(2017) 1243–1253. https://doi.org/10.1007/s00419-017-1246-9.

15. F. Zhou, Z. Zhang, D. Wua, H. Zhu, *An analytical model for predicting the lateral-torsion coupling property of laminated rubber bearings*, J. Sound Vib. 427 (2018) 1–14. https://doi.org/10.1016/j.jsv.2018.04.018.

16. F.P. Beer, E.R. Johnston, P. Cornwell, B. Self, Vector Mechanics for Engineers: Dynamics, eleventh ed., McGraw-Hill, New York, 2015.

17.W.S. Slaughter, The Linearized Theory of Elasticity, Birkhauser, Boston, 2002. https://doi.org/10.1007/978-1-4612-0093-2

18. Segovia, E.; Torres, J.; Carbajo, J.; Ramis, J."Estudio analítico lineal del movimiento de losa cuadrada rígida sobre lámina flexible de pequeño espesor pegada al suelo". Tecniacústica 2014.Murcia, SPAIN: 2014, ISBN ISBN: 978-84-87985-25-6
19. Enrique Segovia; Jeniffer Torres; Jesús Carbajo; Jaime Ramis. Determinación de los parámetros elásticos de un material a partir de las medidas de aceleración del ensayo experimental utilizado para medir las rigidez dinámica". Tecniacústica 2014.Murcia, SPAIN: 2014, ISBN ISBN: 978-84-87985-25-6