Acousto-optic sensing of the sound field in a lightly damped room

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ABSTRACT

Microphone arrays are a powerful means to capture and analyse the spatio-temporal properties of a sound field. The measurement principle is based on a discrete (point-wise) sampling of the sound field, which makes it ill-suited for measuring over large regions of space, e.g. in rooms or other three-dimensional domains of considerable extent. In this study we introduce a method to measure the sound field in a room using the acousto-optic effect. The sensing principle exploits the fact that acoustic pressure fluctuations induce changes in the refractive index of air, and this makes it possible to sense the acoustic field from optical laser measurements. This is inherently a spatial sensing method that enables to capture the sound field over an extended region of space. Furthermore, the sound field can be measured remotely, reaching locations of the room that are not easily accessible by conventional means. In this study we introduce the reconstruction method and assess the quality of the estimated sound field.

Keywords: Acousto-optic, microphone arrays, sensing methods, signal processing, tomography
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1. INTRODUCTION

The ability to measure or predict the spatio-temporal properties of the sound field in a room is valuable, as it enables to analyze and characterize its properties. In situations where it is of interest to capture or predict the sound field over a large three-dimensional aperture, conventional measurement methods (either with microphone arrays or sequential phased measurements) require an unfeasible experimental effort. As a result, the characterization of the sound field is typically restricted to small spatial apertures. To this end, it seems necessary to consider alternative measurement principles that are better suited for measuring a sound field over an extended region of space. Particularly, the the acousto-optic effect, i.e. interaction between sound and light, seems significant, since it can be used as a volumetric sensing method. The acosuto-optic effect describes the process in which light propagating through a medium is influenced by the presence of sound (and viceversa). The refractive index of the medium changes with density and temperature, thus in the presence of sound, the speed of light will change depending on the acoustic pressure field. [1, 2] This phenomenon can be used to sense an acoustic pressure distribution in space.

The diffraction of light caused by sound waves was theoretically predicted in 1922, [3] and first observed experimentally in 1932. [4, 5] Since the 1960s many applications using the acousto-optic interaction emerged, e.g. in optics, acoustics (in air and water) and ultrasound. [6] Over the last couple of decades, an interferometic method based on laser Doppler vibrometry has received significant attention. The method (sometimes referred to as *refracto-vibrometry*) measures phase shifts of laser light induced by local changes of the refractive index, and links them to the acoustic pressure. Originally, it was applied to the visualization of ultrasonic fields, [7–13] and later for sensing and visualizing sound fields in the audible frequency range [14–27] or for estimating sound absorption [28, 29] and diffusion. [30]

To date, the use of acousto-optic sensing methods has been resticted to measurements in controlled laboratory conditions, rather than used in real in situ measurements. This is partly due to technical limitations, and to a large degree due to restrictions derived from the reconstruction methods used. Typically, the existing reconstruction methods have made use of explicit transforms, as commonly seen in computerized tomography (e.g., the well-known filtered back-projection algorithm requires measurements of the sound field from every direction along a set of parallel or fan-shaped beams). [31] These constrains on the measurement configuration have restricted the applicability of acousto-optic tomography.

In this study, we introduce and examine an acousto-optic sound field reconstruction method, formulated as an algebraic reconstruction problem, which does not require regular scanning geometries (unlike e.g. the filtered back-projection method), and thus is suitable for measurements in real complex spaces. As a first study on the subject, we analyze the reconstruction potential based on numerical data, to assess the potential of the method for capturing complex sound fields in a room.

2. THEORY

2.2.1. Acousto-optic interaction

Acoustic pressure fluctuations p can be assumed to be an adiabatic phenomena,

$$\frac{p + p_0}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma}, \tag{1}$$

where p_0 is the static atmospheric pressure, ρ_0 and ρ are the densities of the static and perturbed medium, and γ is the heat capacity ratio (or ratio of specific heat). The index of refraction of a medium is defined as the ratio between the speed of light in vacuum and the medium $n = c_\ell / c_m$. Density and index of refraction are related by the Gladstone-Dale relation [1]

$$n = \rho K + 1, \tag{2}$$

where K is a constant characteristic of the medium. Combining Eqs. 1 and 2, the refractive index is

$$n = (n_0 - 1)\left(\frac{p + p_0}{p_0}\right)^{1/\gamma} + 1 \tag{3}$$

where n_0 is the refractive index under static conditions. Equation 3 can be approximated by its truncated Taylor expansion [22] assuming $|p| << p_0$ as in general linear acoustics,

$$n \approx n_0 + \frac{n_0 - 1}{\gamma p_0} p. \tag{4}$$

Equation 4 is important, as it describes the relation between acoustic pressure and refractive index, i.e. the acousto-optic interaction.

2.2.2. Acousto-optic sensing method

The index of refraction along laser beams can be captured using a laser Doppler vibrometer (LDV). Let consider a sound field $p(\mathbf{r}, t)$ inside a three-dimensional domain Ω ($\mathbf{r}$ indicates position and t is the time variable). A laser beam is emmited from a transmitter and sent through the domain Ω. The beam is then back scattered at the boundary of the domain (e.g. a rigid wall), and collected again at the transmitter. The phase of the back scattered light ϕ depends on the beam optical path [15]

$$\phi(L, t) = k_\ell \int_0^L n(\mathbf{r}, t; l) \mathrm{d}l, \tag{5}$$

where L is the length of the beam path, l is an integration variable along the path, and k_ℓ is the wavenumber of the laser light. The output of a LDV is proportional to the rate of change of ϕ,

$$v(L, t) = \frac{1}{k_\ell n_0} \frac{\mathrm{d}}{\mathrm{d}t} \phi = \frac{\mathrm{d}}{\mathrm{d}t} L(t) + \frac{n_0 - 1}{\gamma p_0 n_0} \frac{\mathrm{d}}{\mathrm{d}t} \left(\int_0^L p(\mathbf{r}, t; l) \mathrm{d}l \right) \tag{6}$$

where the second expression is obtained using Eq. 4. Assuming a fixed boundary where the beam is back scattered, the term $\frac{\mathrm{d}}{\mathrm{d}t} L$ in Eq. 6 is zero. The frequency-domain LDV measurement of the acouto-optic interaction is then

$$V(L, \omega) = \frac{n_0 - 1}{\gamma p_0 n_0} \mathrm{j}\omega \int_0^L P(\mathbf{r}, \omega; l) \mathrm{d}l, \tag{7}$$

where $P(\mathbf{r}, \omega)$ is the frequency-domain sound field.

A key step in the proposed method is the expansion the sound field as a superposition of N plane waves, i.e. expand the pressure field into a plane wave basis in Ω, [32]

$$P(\mathbf{r}, \omega) \approx \sum_{n=1}^{N} X_n \mathrm{e}^{\mathrm{j}\mathbf{k}_n \cdot \mathbf{r}^{\mathrm{T}}}, \tag{8}$$

where X_n is the complex amplitude of the n^{th} wave, and the exponential term accounts for its propagation. The direction of propagation of each wave is given by the wavenumber vector $\mathbf{k}_n$. Only propagating waves are considered in this study, i.e. $|\mathbf{k}_n| = \omega/c$, [33] where c is the speed of sound in the medium. An expression for the plane wave expansion using LDV measurements is obtained by replacing Eq. (8) in Eq. (7) and applying the sum rule in integration,

$$V(L, \omega) = \frac{n_0 - 1}{\gamma p_0 n_0} \mathrm{j}\omega \sum_{n=1}^{N} \left(X_n \int_0^L \mathrm{e}^{\mathrm{j}\mathbf{k}_n \cdot \mathbf{r}^{\mathrm{T}}} dl \right). \tag{9}$$

The sound field can be measured with M beams following various paths l_m. Equation 9 can then be expressed algebraically as

$$\mathbf{v} = \mathbf{H}\mathbf{x}, \tag{10}$$

where $\mathbf{v} \in \mathbb{C}^M$ are the LDV measurements, $\mathbf{x} \in \mathbb{C}^N$ are the complex amplitudes of the plane waves, and the sensing process is represented by the matrix $\mathbf{H} \in \mathbb{C}^{M \times N}$, with element $\mathbf{H}_{mn} = \frac{n_0-1}{\gamma p_0 n_0} \mathrm{j}\omega \int_0^{L_m} \mathrm{e}^{\mathrm{j}\mathbf{k}_n \cdot \mathbf{r}^{\mathrm{T}}} dl_m$. The plane wave complex amplitudes are estimated by fitting $\mathbf{x}$ to the measurements in the least-square sense,

$$\tilde{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{C}^N}{\arg\min} \, \|\mathbf{H}\mathbf{x} - \mathbf{v}\|_2^2 + \lambda^2 \|\mathbf{x}\|_2^2. \tag{11}$$

Generally, measurements are contaminated with noise ($\mathbf{v} = \mathbf{v}^{\mathrm{exact}} + \epsilon$), and a regularization term must be included in order to arrive to a stable solution. In this study, conventional Tikhonov regularization is applied, and the regularization parameter λ is chosen by generalized cross-validation. [34] Estimations of the pressure at any S positions in Ω can be obtained from $\tilde{\mathbf{x}}$:

$$\tilde{\mathbf{p}} = \mathbf{G}\tilde{\mathbf{x}}, \tag{12}$$

where $\mathbf{G} \in \mathbb{C}^{S \times N}$ is the reconstruction matrix, with element $\mathbf{G}_{sn} = e^{\mathrm{j}\mathbf{k}_n \cdot \mathbf{r}_s^{\mathrm{T}}}$.

3. RESULTS

The reconstruction of the sound field in a room using the proposed acousto-optic sensing method from simulated data is examined in this section. Laser measurements $V(L, \omega)$ inside a room are simulated for $M = 64$ beams. The beam paths, shown in Fig. 1, cover one plane ($z = 1.5$ m) of the room. The laser transmitters are placed in each of the four corners, and the light is back-scattered at the opposite walls. The room, with dimensions $4.38 \times 3.29 \times 2.97$ m, is assumed to be lightly damped. The sound field is calculated via the solution of the wave equation for a rectangular enclosure. [35] The laser measurements are approximated by integrating the pressure along the beam paths numerically. White Gaussian noise is added to the simulated laser measurements to achieve SNR = 20 dB.

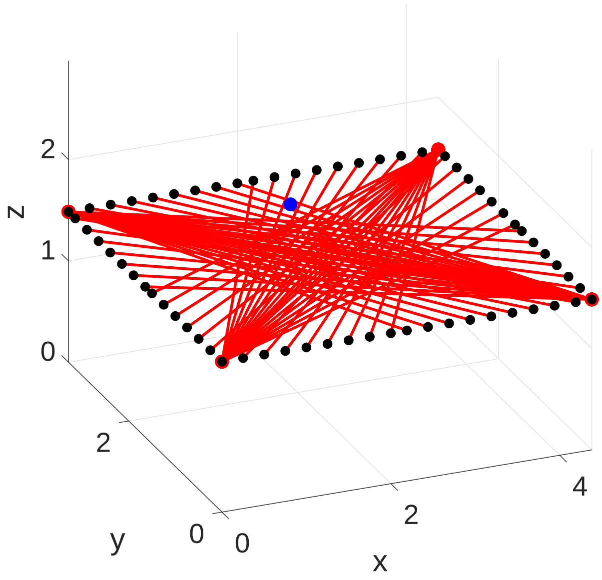

Figure 1: Laser beam paths. Red dots: laser transmitters. Black dots: points on the boundary where the beams are back scattered. Blue dot: point for evaluation across frequency. Dimensions in meters.

Figure 2a shows the reference sound pressure field, calculated at the measurement plane at 200 Hz. Figure 2b shows the reconstructed sound field from the 64 laser measurements of Fig. 1. Figure 2c shows the relative error (difference between reference and reconstruction over the reference pressure) in dB. There is a good agreement between reference and reconstruction. The relative error is below 0 dB in most of the plane. The qualitative reconstruction of the sound pressure field is very good. There are quantitative differences, as shown in Fig. 2c, mostly near the nodal planes, where the sound pressure field changes rapidly. The reconstruction values are slightly shifted, mostly due to small errors in the estimation of the plane wave coefficients (i.e. the solution of Eq. 11). The estimation errors result in slight shifts of the interference pattern between the sound waves in the room, thus shifting the nodal planes.

Figure 3 shows the reconstruction of the sound field at 400 Hz. Figure 3a shows the reference sound field, Fig. 3b shows the reconstructed sound field from 64 laser measurements and Fig. 3c shows the relative error. It should be noted that the Schroeder frequency of the room is 400 Hz, indicating that the sound field is very complex, due the high modal overlap and the interference between modes. The point-wise error of the estimated field is larger in this case, although the overall spatial features of the sound field are correctly captured, leading to a fair qualitative characterization.

Figure 4 shows the reconstructed pressure at a single point, across frequency, between 50 H and 500 Hz. The position of the point is 2.32, 2.73, 1.5 m, as indicated in Fig. 1. Good agreement is achieved for frequencies below 300 Hz. At higher frequencies the reconstruction degrades, although some of the room modes are correctly recovered, and the estimated frequency response function is qualitatively fair. It should be noted that the Shroeder frequency of the room is at 400 Hz, so it is not surprising that the deterministic estimation shown in the figure deviates from the exact one.

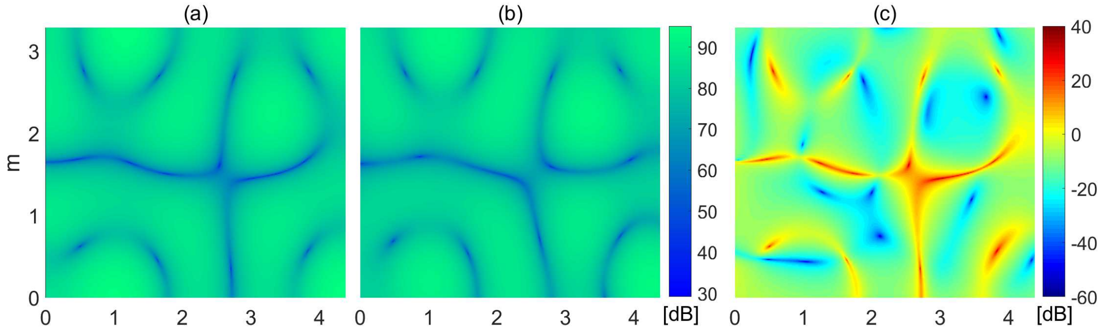

Figure 2: Sound field in the room at 200 Hz (plane at z=1.5 m). (a) reference sound field, (b) reconstruction from $M = 64$ *laser measurements (c) relative error.*

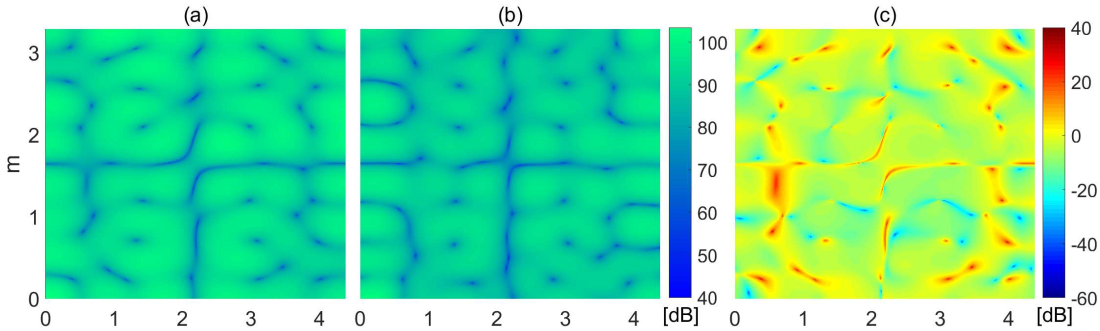

Figure 3: Sound field in the room at 400 Hz (plane at z=1.5 m). (a) reference sound field, (b) reconstruction from $M = 64$ *laser measurements (c) relative error.*

4. DISCUSSION

The presented results show the measurement and reconstruction of the sound field at a single plane for the sake of visualization clarity. Naturally, the sound field can be measured and reconstructed in three-dimensions using the proposed acousto-optic sensing method.

The LDV measurements of the acouto-optic interaction are proportional to $\frac{n_0-1}{\gamma p_0 n_0} \approx 2.68 \cdot 10^{-9}$ (Eq. 7). Therefore, very high pressure levels are required to achieve a sufficient SNR when dealing with real measurements inside a room. In addition, the measurements are proportional to the line integral of the pressure along the beams. As a consequence, zero (or close to zero) values are measured for waves travelling in the same direction as the beam.

It should be noted that vibrations of the scanned surfaces can bias the acousto-optic measurements: if the surfaces that back-scatter the laser beam are vibrating, the assumption in Eq. 7 is not valid. The measurement is in this case the combination of the the acousto-optic interaction and the actual velocity of the surface (in fact, measuring the velocity of surfaces is the original purpose of LDV devices). The influence of surface vibrations will be assessed in future work, e.g. monitoring the vibration of the walls.

The plane wave model used in the proposed method makes it possible to estimate physically meaningful quantities—pressure, particle velocity, active and reactive intensity—, as opposed to other algebraic reconstruction approaches, which are closer to image reconstruction methods.

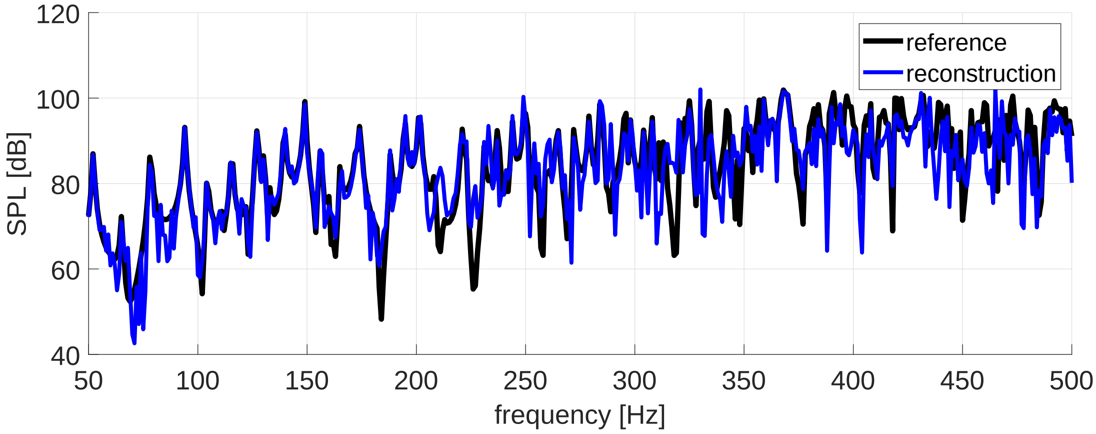

Figure 4: Sound field in the room at position 2.32,2.73,1.5. Reference and reconstructed sound pressure level across frequency

5. CONCLUSION

This study introduces a reconstruction method based on the acousto-optic effect that makes it possible to measure the sound field in the volume enclosed by a room. The method is posed as an algebraic reconstruction, therefore enabling to reconstruct the sound field from arbitrary measurement configurations, circumventing the need for restrictive scanning patterns. This initial study introduces the reconstruction method, and tests its validity using numerically simulated data in a rectangular room. The results indicate that the reconstruction principle is suitable, and will be tested experimentally in the future.

[1] W. Merzkirch. *Flow visualization*. Academic Press, London, 1974. Chap. 3: Optical methods for compressible flows.

[2] H. Kleine, H. Grönig, and K. Takayama. Simultaneous shadow, schlieren and interferometric visualization of compressible flows. *Opt. Lasers Eng.*, 44:170–189, 2006.

[3] L. Brillouin. Diffusion de la lumièrie et des rayons X par un corp transparent homogene. *Ann. Phys.*, 9(17):88–122, 1922.

[4] P. Debye and F. W. Sears. On the scattering of light by supersonic waves. *Proc. Nat. Acad. Sci.*, 18(6):409–414, 1932.

[5] R. Lucas and P. Biquard. Optical properties of solids and liquids under ultrasonic vibrations. *J. Phys. Rad.*, 7(3):464–477, 1932.

[6] I. C. Chang. *Handbook of optics, Volume II*. McGraw-Hill, New York, 1995. Chap. 12: Acousto-optic devices and applications.

[7] X. Jia, G. Quentin, M. Lassoued, and J. Berger. Quantitative measurement of ultrasonic pressures with an optical heterodyne interferometer. *Proc. IEEE Int. Ultrason. Symp.*, pages 1137–1140, 1991.

[8] X. Jia, G. Quentin, and M. Lassoued. Optical heterodyne detection of pulsed ultrasonic pressures. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 40(1):67–69, 1993.

[9] K. Nakamura. Sound field measurements through the acousto-optic effect of air using laser doppler velocimeter. *Proc. IEEE 4th Pacific Rim Conference on Lasers and Electro-Optics*, pages 154–155, 2001.

[10] J.P. Remenieras, O. B. Matar, S. Calle, and F. Patat. Acoustic pressure measurement by acousto-optic tomography. *Proc. IEEE Int. Ultrason. Symp.*, pages 154–155, 2001.

[11] K. Nakamura, M. Hirayama, and S. Ueha. Measurements of air-borne ultrasound by detecting the modulation in optical refractive index of air. *Proc. IEEE Int. Ultrason. Symp.*, pages 609–612, 2002.

[12] R. Balek and Z. Slegrova. A comparison of acoustic field measurement by a microphone and by an optical interferometric probe. *Acta Polytech.*, 42:13–17, 2002.

[13] J. M. Buick, J. A. Cosgrove, P.-A. Douissard, C. A. Greated, and B. Gilabert. Application of the acousto-optic effect to pressure measurements in ultrasound fields in water using a laser vibrometer. *Rev. Sci. Instrum.*, 75(10):3203–3207, 2004.

[14] L. Zipser and H. Franke. Visualization and measurement of acoustic and fluidic phenomena using a laser-scanning vibrometer. *Fifth International Conference on Vibration Measurements by Laser Techniques: Advances and Applications*, pages 192–2198, 2002.

[15] L. Zipser, H. Franke, E. Olsson, N.E. Molin, and M. SjÃűdahl. Reconstructing two-dimensional acoustic object fields by use of digital phase conjugation of scanning laser vibrometry recordings. *Appl. Opt.*, 42(29):5831–5837, 2003.

[16] A. R. Harland, J. N. Petzing, and J. R. Tyrer. Nonperturbing measurements of spatially distributed underwater acoustic fields using a scanning laser doppler vibrometer. *J. Acoust. Soc. Am.*, 115(1):187–195, 2004.

[17] Y. Oikawa, M. Goto, Y. Ikeda, T. Takizawa, and Y. Yamasaki. Sound field measurements based on reconstruction from laser projections. *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, pages 661–664, 2005.

[18] K. Tatar, E. Olsson, and F. Forsberg. Tomographic reconstruction of 3d ultrasound fields measured using laser vibrometry. *International Conference on Experimental Mechanics*, pages 337–338, 2007.

[19] L. Zipser, Seelig H. D, and H. Franke. Refracto-vibrometry - a novel method for visualizing sound waves in transparent media. *Proc. European Conference on Noise Control*, pages 1997–2001, 2008.

[20] E. Olsson and F. Forsberg. Three-dimensional selective imaging of sound sources. *Opt. Eng.*, 43(3):1–8, 2009.

[21] L. Zipser, Seelig H. D, and H. Franke. Refracto-vibrometry for visualizing ultrasound in small-sized channels, cavities and objects. *Proc. IEEE Int. Ultrason. Symp.*, pages 2588–2591, 2009.

[22] A. Torras-Rosell, S. Barrera-Figueroa, and F. Jacobsen. Sound field reconstruction using acousto-optic tomography. *J. Acoust. Soc. Am.*, 131(5):3786–3793, 2012.

[23] E. Fernandez-Grande, A. Torras-Rosell, and F. Jacobsen. Holographic reconstruction of sound fields based on the acousto-optic effect. *Proc. Internoise*, 2013.

[24] A. Torras-Rosell, O. Lylloff, S. Barrera-Figueroa, and F. Jacobsen. Reconstruction methods for sound visualization based on acousto-optic tomography. *Proc. Internoise*, 2013.

[25] R. Malkin, T. Todd, and D. Robert. A simple method for quantitative imaging of 2d acoustic fields using refracto-vibrometry. *J. Sound Vib.*, 333:4473–4482, 2014.

[26] K. Bertling, J. Perchoux, T. Taimre, R. Malkin, D. Robert, A. D. Rakic, and T. Bosch. Imaging of acoustic fields using optical feedback interferometry. *Opt. Express*, 22:30346–30356, 2014.

[27] K. Yatabe, K. Ishikawa, and Y. Oikawa. Acousto-optic back-projection: Physical-model-based sound field reconstruction from optical projections. *J. Sound Vib.*, 394:171–184, 2017.

[28] C. Vuye, S. Vanlanduit, and P. Guillaume. Accurate estimation of normal incidence absorption coefficients with confidence intervals using a scanning laser doppler vibrometer. *Opt. Lasers Eng.*, 47:644–650, 2009.

[29] F. Mbailassem, Q. Leclere, E. Redon, and E. Gourdon. Experimental analysis of acoustical properties of irregular cavities using laser refracto-vibrometry. *Appl. Acoust.*, 130:177–187, 2018.

[30] R. Jackett, B. Piper, and R. J. Hughes. Characterization of acoustic diffusion using refracto-vibrometry. *J. Sound Vib.*, 392:70–76, 2017.

[31] A. C. Kak and M. Slaney. *Principles of computerized tomographic imaging*. IEEE, New York, 1988. Chap. 3: Algorithms for reconstruction with nondiffractive source.

[32] S. A. Verburg and E. Fernandez-Grande. Reconstruction of the sound field in a room using compressive sensing. *J. Acoust. Soc. Am.*, 143(6):3770–3779, 2018.

[33] E. G. Williams. *Fourier acoustics - sound radiation and nearfield acoustic holography*. Academic Press, San Diego, 1999. Chap. 2: Plane waves.

[34] P. C. Hansen. *Discrete inverse problems - insight and algorithms*. SIAM, Philadelphia, 2010. Chap. 5.

[35] F. Jacobsen and P. M. Juhl. *Fundamentals of General Linear Acoustics*. Wiley, London, 2013. Chap. 8: Sound in enclosures.