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NOISE CONTROL FOR A BETTER ENVIRONMENT

Airborne sound source characterization for railway noise predictions - based on vibration measurements and numerical simulations

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ABSTRACT

Within the European rail initiative Shift2Rail, funded by the EU, the FINE1 project aims to improve state-of-the-art noise modeling for railway systems. Current prediction tools are able to compute the sound pressure field distribution around full rail vehicles. Once the pressure field distribution of a source has been determined and the transfer functions ($TF = SPL_{SpatialDistrib.} - SWL_{SoundSource}$) are estimated, the resulting frequency-dependent spatial pressure distribution for sources specified by sound power can directly be quantified and visualized as blocked pressure. In these TF calculations the sources are predominantly integrated as monopoles: a solution has to be found for the implementation of the directional characteristic. The necessary input for some sources, at least with direct sound pressure/intensity measurements, is difficult to obtain (due to unavailability of anechoic/reverberation rooms). One feasible solution for vibro-acoustic sources, in principle as standard available, is the determination of airborne sound power by use of vibration measurements with 'fixed/adequate' radiation.

The paper starts with a simulation of an acoustic measurement for an exemplary gearbox at a defined operating point. Besides the required airborne sound power and sound directivity of the gearbox, the simulated measurement results provide also a full vibration data set for the complete outer surface. The objective of this investigation is to 'measure' the 'same' sound power and directivity with a limited, selected number of vibration sensors. The used tools, software as well as hardware, shall be available within the framework of the railway industry.

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1. INTRODUCTION

The objective of this investigation is to determine the emitted airborne sound power and directional characteristic of a gear unit based on a vibration measurement with as few vibration sensors as necessary. This is a challenge for computational tools like FEM or BEM since these methods require vibration information for all modelled elements but do not offer many technical hints how to expand the vibration from just a few points to all elements.

Right from the beginning, modal analysis is the only promising method to combine the ‘all elements’ requirement with the initial condition of only a few measurement points. It is a system analysis executable as Experimental Modal Analysis or Finite Element Analysis (EMA/FEA) and a method that contains essential information for the radiation problematic apart from the operational excitation. Every EMA starts with a limited number of measurement points and an excitation, it looks like any operational deflection shape (ODS). The major difference being the kind and knowledge of excitation. That leads to the concept of the operational modal analysis (OMA) by trying to avoid the additional efforts of an EMA. The complete solution is given here with the 4 necessary steps in advance:

1. Computational modal analysis of the gearbox housing;
2. Operating vibration measurements on the housing with a limited, selected number of measurement points;
3. Measurement expansion using the linear complex spectra and the FE modal analysis;
4. Calculation of the radiated sound power and directional characteristics.

The preliminary steps like the OMA are documented here and used to assure the quality of data and procedure.

The question whether the sound power or sound pressure is the right way to quantify acoustic sources, is already decided a long time ago. Whenever there a need to transfer the data to other measuring distances or into other rooms, it is required to use the sound power. This question is of some relevance here because the power radiated by the modes cannot be in general added together for a valid calculation of the total radiated sound power. The superposition principle is only valid for the sound pressure but not for the sound power.

Gear noise is an important sound source and gains extra importance for manufacturing quality due to the correlation between dynamic loading on the teeth for a life cycle and the identification of manufacturing defects [1]. The gear unit vibration signature is in general dominated by spectral peaks. The ‘total acoustics system’ is certainly not limited to the gear unit but ‘includes the prime mover, driven equipment, gear unit mounting, foundation and acoustic environment’ (see [2], [3]) as part of the system.

Most of sound power estimates are based on microphone sound pressure measurements using defined microphone configurations or arrays, sound intensity measurements and sound energy measurements. Sensor combinations with hotwire -measuring particle velocity- configurations are also possible. They are based on sound field assumptions (direct field, near and far field) or measure field oriented, e.g. near field holography. All common acoustic power measurements require a different degree of technical, environmental or organizational conditions, which are not always available.

The current investigation considers the case of vibro-acoustic sound sources where a direct air-borne sound power measurement is not possible or considered too inaccurate. One feasible solution for vibrating sources –as standard available [4] [5] - is the

determination of airborne sound power by use of vibration measurements with ‘fixed/adequate’ radiation efficiency (σ). ‘Fixed’ radiation efficiency refers to an assumed radiation efficiency equal to one ($\sigma = 1$). ‘Adequate’ radiation efficiency refers to an available value, whether by calculation or measurement. The use of the vibration velocity assumes that the sound power is proportional to the spatially averaged root mean squared normal velocity and the area of the vibrating gear housing surface. This is not the case for e.g. the aero-acoustic sources.

‘The accuracy with which radiation efficiencies can be measured is ± 0.5 dB, under the most favorable circumstances. This accuracy is limited primarily by how well one can determine the mean-square velocity’ [6]. It means that the velocity distribution is of major importance for a well-quantified radiation efficiency. The more the velocity varies from measure point to measure point due to inhomogeneous properties or excitations (e.g. local mass, stiffness, damping or force distribution) the more difficult becomes the task to estimate a useful spatial velocity average or the respective radiation efficiency. Cremer & Heckl provide the following conclusion: ‘Measurements of the radiation efficiency are meaningful only if the velocity of the radiating area is relatively uniform’ [6].

Singh [7] experimentally determined the radiation efficiency with sound intensity and ideal models (monopole, dipole, flat plate, or cylinder) and computational acoustics (FEM/BEM) to predict the radiation efficiency for a gear housing (top plate). Singh comes to the following conclusions: ‘predictions of the housing characteristics were only partially successful’. He summarizes further: The FE model is able to compute the vibration modes, the structural response of the gearbox is predicted if the dynamic excitation is adequately simulated. A BEM can predict the vibration response if the vibration characteristics is known. Ideal models cannot predict the variation of the acoustic response since the modal characteristics of the gearbox are not simulated, general trends and the model usage are difficult to justify for these geometries. Besides the inaccurate modeling of the excitation, coupling of modes seems to be a probable cause for result discrepancies also [7].

Elliott & Johnson [8] compare the calculation of the radiated total sound power in terms of amplitudes of structural modes with the formulation in terms of velocity amplitudes of elemental radiators. In both formulations depends the radiation of single elements (structural mode and elemental radiator) on other elements as well. They name independent radiating velocity distributions ‘radiation modes’ and show that these can be computed as the eigenvectors of an elemental radiation resistance matrix. Elliott & Johnson write ‘The most important theoretical result is a quantification of the number of sensors required to accurately predict radiated sound power from measurements on the surface of the structure’ [8]. It seems that these results are so far only used for selected configurations (e.g. baffled, planar) and not implemented in commercially available tools.

2. PHENOMENOLOGY AND MEASUREMENT STRATEGY

The following section is based on Cremer & Heckl thought-experiments about radiation problems from arbitrarily shaped structures for a given particle velocity distribution [9]. It is assumed that the normal velocity can be measured on the whole gearbox housing, the translational components -due to air viscosity- are thereby neglected. If the directivity is not of interest but only the radiated airborne power, several simplified methods like ‘Rayleigh Method’ are available, which compute the radiating gearbox with small monopoles. Thereby, gear housing is thought removed and only the vibrating structure is present, but not the diffracting or scattering part. Other methods like FEM, BEM or the Equivalent Source Method include the housing and are able to provide

the diffraction part but they require a sufficiently small distance between the vibrating elements. According to the rule of thumb given by Cremer & Heckl, the area dimensions of the vibrating elements shall be smaller than one-third of the distance between regions vibrating in opposite phase. Furthermore, these dimensions shall be smaller than a sixth of the radiated wavelength. Cremer & Heckl summarize the discussion with the remark, that the complete sound field description of arbitrarily shaped structures with a defined particle velocity distribution is for infinitesimal small surface elements a discretized form of the rigorous solution of the radiation problem.

All these methods, therefore, require the use of the maximum number of vibration sensors distributed over the whole gear housing. The number of available sensors becomes then significant for the usable, valid frequency range. After the vibration data are mapped on a FEM/BEM model, the radiated sound power as well as the directivity can be computed for the valid frequency range. However, the use of a large number of sensors contradicts the current objective of this investigation.

A system analysis based on operational gear unit measurement with a reduced number of sensors leads to a reduced, but valid and usable number of vibration positions. The ODS provides the complex deflection of the structure at a single frequency and a defined operating point. It provides with the displacement, velocity or acceleration measurement a combined motion for defined points on a gear unit. The measurement enables calculation of an OMA (see section 5) via frequency response functions (ODS-FRF) and provides an estimate of the modal parameters. This enables in turn the validation of a Finite Element Analysis (FEA) and an expansion of the measurement with a valid computational FE model, which has much higher DOF. The OMA provides confidence for the validity of experimental measurement and the FEA. The validity of the mode shapes and measurement are the relevant factors according to

$$\{x_{expanded}\} = \sum ([\Psi] \{MPF\}) \quad \text{Equation 1}$$

$$\{MPF\} = [\Psi]^{-1} \{x_{meas}\} \quad \text{Equation 2}$$

where x is the displacement, MPF the modal participation factor and Ψ the modal matrix.

3. SIMULATED MEASUREMENT

Figure 1 shows the measurement configuration: the gearbox (surface ca. 1.1 m²) and a very fine field point mesh (1865 microphones) for the sound power and directivity calculations. A fine ISO standard field point mesh with 38 field points is also used for the sound power calculation (not shown here). Using the structural mesh (336168 nodes, 180997 elements; 1016742 DOF) 79 modes were computed in the frequency range up to 5 kHz. The modal computation is conducted using VirtualLab (Samcef) and checked using Simcenter (Nastran) and Altair HyperWorks (Optistruct). The structural mesh configuration is excited at 4 nodes (each 3 DOF) at an operation point with 3000 RPM. VirtualLab BEM (9835 nodes, 19666 elements) and FEM (53009 nodes, 243541 tetra4-elements, Automatic Matched Layer-AML) models are used for the acoustics calculations. The sound power and directivity results in the current paper are computed with FEM (AML). The frequency range of interest reaches from 10 Hz to 2818 Hz in 1 Hz steps (intended for third-octave limits). The acoustic results are presented for the frequency range 10 Hz to 2 kHz.

Figure 2 displays the sound power, mean quadratic velocity and the radiation efficiency for a configuration without and with damping. The modal damping is shortly described with 'ca. 1 %', but it changes between 0.5 % and 1.7 % (1st mode 0.5 %; 2nd

1.4 % etc.). The average damping up to 2 kHz is ca. 0.8 %. The plot of the radiation efficiency includes two additional calculations with unit normal velocity excitation of the gear housing outer surface using Direct BEM and FEM (AML) to confirm the radiation efficiency problematic investigated by Singh [7]: there are major differences even with the use of the original geometry.

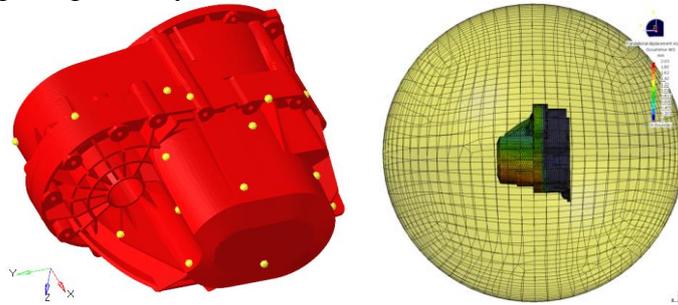


Figure 1: FEA model with selected 35 MPs resp. 105 DOF (left), field points for the SWL and directivity calculations

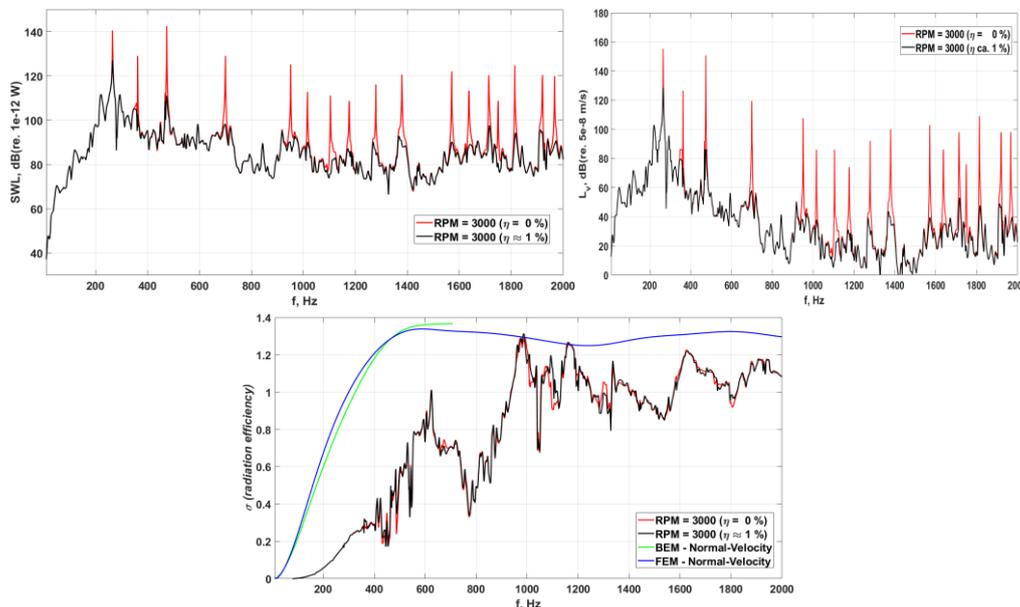


Figure 2: Sound power level (left), velocity level (right) and radiation efficiency (bottom)

4. SYSTEM ANALYSIS (POINT SELECTION)

A limited number of measurement points requires a selection strategy because measurement at nodal points reduce the overall available information. Decisions about a minimal amount of vibration sensors, their position and direction face many technical challenges not only due to placement at nodal points. Local modes, coupled modes, repeated roots, strong local damping/absorption or dominant forced excitation provide further challenges. Nevertheless, all modes have to be detected for a full system information. The reduced number of measurement points does not promote engineering understanding, plausibility and intuition due to the reduced visual information and requires the extension of mathematical means.

The pre-test analysis shall assure a risk reduction as well as an optimal sensor usage. The following proposals are mainly the essence of an article series provided by Vibrant Technology, in particular [10]. The Open Source literature by Prof. Avitabile offers also helpful explanatory technical hints. Some commercial tools like Siemens LMS-VirtualLab/SimCenter3D, FEMTools, and ME'scope provide pre-test tools usable for

measurement point optimization. Almost all pre-test functions considered here are determined with ME'scope:

1. Modal Assurance Criterion (measurement point selection)
2. Counting peaks ($Y_{\text{DrivePoint}} / Z_{\text{DrivePoint}}$, nodal points, spatial mode description)
3. Driving point mobility/impedance (degree of inhomogeneities)
4. Shape product (nodal points)
5. Reference point selection
6. Modal summation (SUM)
7. Modal indicators (CMIF, MMIF)

All points above improve the quality of an EMA or OMA. The following sections provide a short description of these topics.

4.1 MODAL ASSURANCE CRITERION (MAC)

The MAC is similar to the spectral coherence and quantifies the degree of linearity between two modal vectors. The MAC values are limited between zero and one. There are several reasons for low MAC values like non-stationarity, noise, invalid modal parameters, non-linear relationship, 'non-orthogonality'. The MAC is considered here for optimal vibrations sensor positioning, validation, and correlation of modal models called 'shape table'. The MAC values are determined with MEscope and Matlab scripts [11].

Figure 3 shows that only a few smart selected points are sufficient to identify the modes (diagonal values = 1). High off-diagonal values indicate that the discrimination between these mode shapes with the selected sensor degree of freedom is not good. The selected single point plays already a significant role (top-left, -right). The bottom-left figure shows that two combined points discriminate the modes even better. Further addition of two other points reduces the off-diagonal values and improves the spatial mode separation. Decisive selected points are a good reference for multiple reference selection. FEA and EMA use the induced forces as input reference for all response signals. Are the input forces not measurable is a response sensor selected as a reference (see section 4.5 and 5).

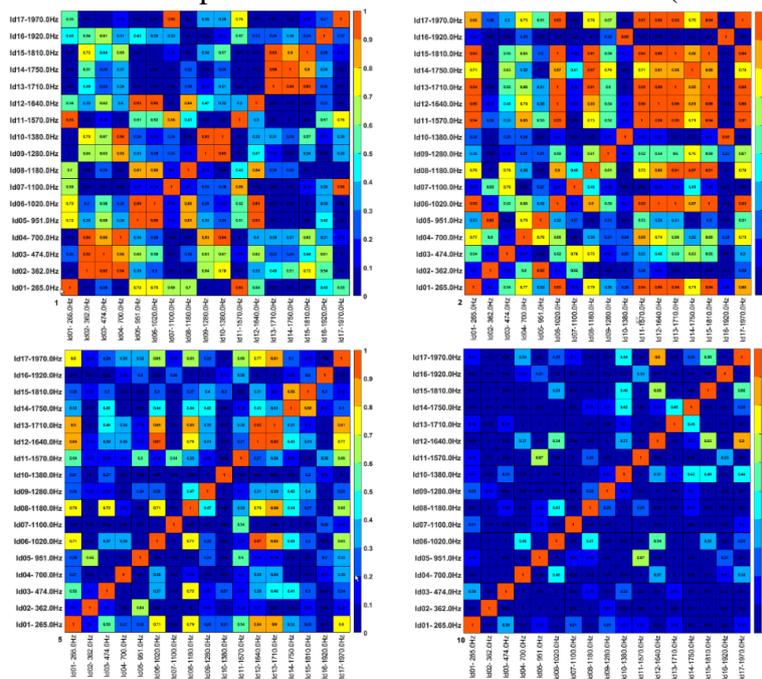


Figure 3: Results of MAC analysis of 35 MPs vs single reference point (top-left), another single reference point (top-right), both reference point (bottom-left) and 4 reference point (bottom-right)

Figure 4 (left) shows the MAC-analysis for the FEM modal analysis of the 35MPs (3 DOF each) and the full modal analysis 8904 MPs (3 DOF each) mapped onto the outer surface of the gearbox housing.

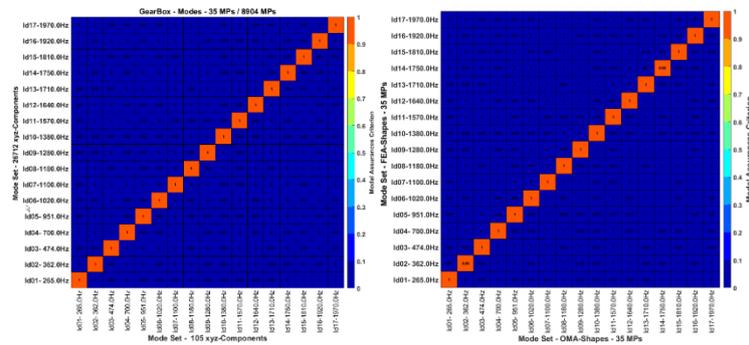


Figure 4: MAC for FEA 35 MPs vs FEA 8904 MPs (left), MAC for OMA 35 MPs vs FEA 35 MPs (right)

4.2 ‘COUNTING PEAKS’

The drive point mobilities are computed using FRF-synthesis in VirtualLab and Altair. ‘Counting peaks’ using the drive point mobilities shown in Figure 5 (left), displays for MP01 all 17 modes of interest with different levels, considering e.g. mode 1 and 12. The second selected MP02 does not display the peaks of mode 1, 3, 8, 12 and 13.

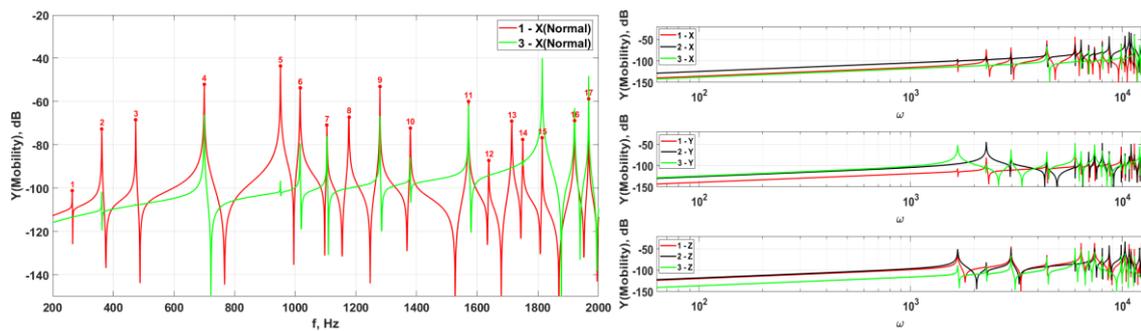


Figure 5: Counting peaks for DOF 1X and 3X (left), mobility of 9 selected MPs, (right)

4.3 DRIVING POINT MOBILITIES/IMPEDANCE

Figure 5 (right) shows also the drive point mobility for a few measurement points. These points are nodes of elements with surface normal vectors in x, y, and z-direction. A mobility plot of all selected measurement points provides a survey about the degree of homogeneity of mass, stiffness, and damping. The drive point mobility (resp. impedance) can be used to analyze the frequency dependent mass and stiffness distribution (not considered here).

4.4 SHAPE PRODUCT (NODAL POINTS)

The Shape Product is a matrix multiplication of all mode shapes of interest and visualizes e.g. nodal information. The Shape Product result is a new shape and can be displayed and animated. The new shape keeps all the original nodal points and provides a good hint for a measurement point selection as well as for reference points. Figure 6 shows a Shape Product of the gear unit. The first single reference point for the OMA is selected from the moving area in the Shape Product (compare Figure 1 and Figure 6).

4.5 REFERENCE POINT SELECTION

For every modal analysis is a selection of excitation and response points necessary. A reference point is a fixed force or response point. A single reference point being very responsive to excited modes of interest provides usually very good results. If more than one reference point is necessary a new decision about the number, location and direction needs to be taken. Without the need of a roving response, i.e. simultaneous measurement. all sensors could be a reference. Since the excitation forces at the operational gear unit cannot be directly measured an ODS-FRF/OMA is done. Each measurement point detects modes differently (s. also Figure 3, Figure 4 and Figure 6).

4.6 MODAL SUMMATION (SUM)

The SUM (s. Figure 6) is a summation function of all FRFs: the remaining peaks are probably modes due to the global activity, the SUM depends on the modal density and spatial separation.

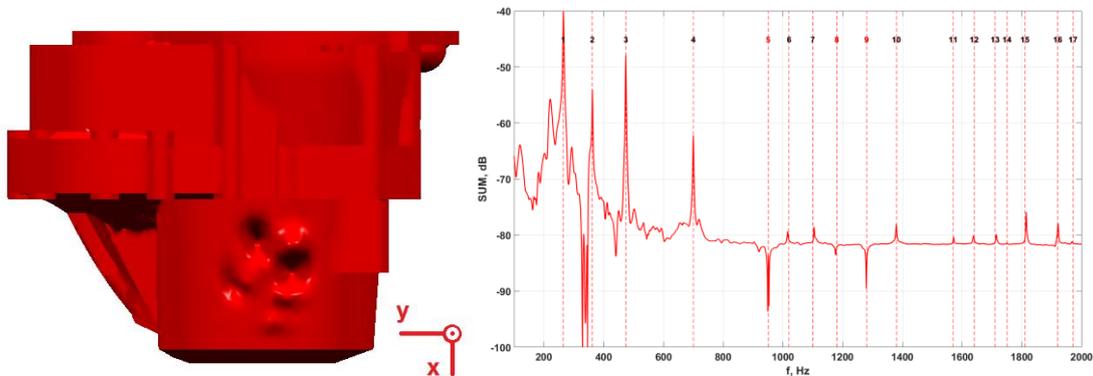


Figure 6: Results of the Shape Product operation (left), SUM (right)

4.7 MODAL INDICATORS (CMIF, MMIF)

Modes are not always easy to identify, due to inactivity, directionality, symmetry or repeated roots. All mode indicator functions support the identification of modes in a different way and accuracy. Some indicators are able to detect modes of repeated roots, for coupled modes, for each reference, for separate modes in a narrow frequency range. A gear unit example of CMIF and MMIF are shown in Figure 7. Since it is already known from the modal analysis (FEA and OMA) that the system has no repeated roots or coupled modes, this could be confirmed with these indicators again.

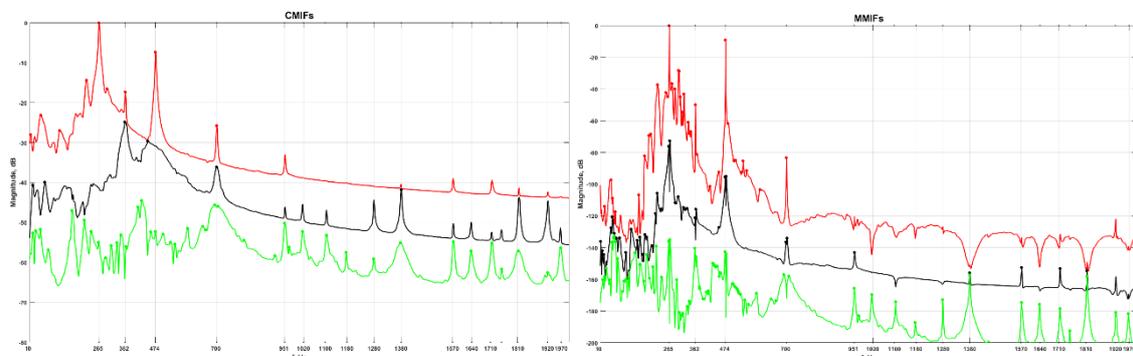


Figure 7: CMIFs (left) and MMIFs (right) for 3 reference points

4.8 OPEN QUESTIONS

Not all the means presented here are used in this investigation to change the selected measurement points again: the procedure is started with a uniform distribution over the vibrating part of the gear housing. The MAC values are used to reduce the off-diagonal values. Since e.g. the Shape Product displayed a limited ‘moving area’, it is directly used for a reference selection. E.g. peak counting, SUM, and multiple references are not used at the beginning of the investigation. The OMA is started and presented with a single reference. For the multiple references and the MMIF are e.g. the MAC values checked again. Since the sensitivity and relevance of these single criteria for the final radiation are unknown, these procedures should be further investigated.

5. OMA (ODS-FRF)

‘Shape’ is defined as the relative position/motion of two or more points, one point being the reference. Two different shapes are used here: operational deflection shape (ODS) and mode shape (MS). The MS is one parameter of modal analysis, the other two are eigenfrequency and damping. MS’s have a unique shape and are an inherent ‘Eigen’ property of the structure and characterize the resonance behavior, but do not have unique values or units. MS’s are defined for linear motions and considered a robust quality and change only with the material (mass, stiffness, damping) or boundary conditions.

Every EMA starts with an ODS. An ODS changes with forces/load, scales and has units. ODS is a combination of forced and resonance behavior and is defined for non-resonating structures as well. ODS is defined for linear as well for non-linear motions. Any forced operational measurement can be considered an ODS if the ‘shape information’ is available by a linear complex spectrum, auto & cross-spectrum, frequency response functions, ODS-FRF or transmissibility. The ODS-FRF is a combination of the cross- and auto-spectrum and as such the basis for an OMA, it provides for each MP the cross-spectrum phase to a reference point and a magnitude of the auto-spectrum response point.

Figure 4 (left) shows MAC values of the MPs35-OMA-and MPs8904. MAC values are of similar order like the values in Figure 4 (right) and guarantee an agreement between computational FE modal analysis and the OMA. An ODS depends not only on the forces but contains usually combinations of all modes and is influenced by the MS as well as eigenfrequencies. An ODS at eigenfrequencies or close to them is dominated by the MS. For further information see [12].

6. RESULTS OF THE MODAL EXPANSION

The method called ‘modal expansion of experimental data’ is already used for the acoustic radiation of an engine block by Guisset & Brughmans in 1995 [13]. The structural analysis (FE 7337 nodes, 6337 elements; 44000 DOF) used 34 mode shapes for the frequency range up to 2.5 kHz. The acoustic radiation is computed with BEM (2592 nodes, 2759 elements). The method is also experimentally validated with 13 vibrations sensors measuring the normal components for a frequency range of up to 3 kHz. The excitation is done with a pneumatic hammer. The acoustics is measured with 18 microphones in a distance of 0.1, 0.3 and 1 m. The validation results are assessed with: ‘extremely good correlation between analysis and test results in shape and amplitude values.’

The only measurement input provided for the expansion procedure is the linear complex response spectra of the 35 selected triaxial MPs (105 DOF, see section 3 and 4),

in short, the ODS. ‘Expansion’ refers to the act of increasing the limited number of measurement points by use of the FEA modal analysis. In ME’scope from Vibrant Technology, this procedure is a single step called ‘measurement expansion’. Measurement expansion increases the number of measurements points using a set of mode shapes with a higher number of DOF. The ME’scope ‘Help’-function notes: ‘If a set of FEA mode shapes with many M#s represents a valid dynamic model of a structure, then expanding an ODS or a set of EMA mode shapes using the FEA mode shapes provides a valid Expanded ODS or set of EMA mode shapes for all of the unmeasured DOFs of the structure represented by the DOFs in the FEA mode shapes.’ The number of vibration sensors and the number of modes is of relevance for the computational process (here 105 DOF/ 17 Modes), an overdetermined equation system is in general of advantage.

In the following section are the original ‘true measurement’ results (the solution of the Helmholtz equation using the original vibration data) compared with the simulation results using the expanded ODS, the ‘measurement expansion’ (the solution with the expanded data). Figure 8 displays the sound power spectra and the level difference, the ratio of the power. The biggest difference appears at 2 kHz. This is the frequency limit of the transferred data.

The velocity level (s. Figure 9) agrees also well and deviates only at a single point due to a small frequency shift at 347 Hz (not visible in the figure). The radiation efficiency (Figure 10) is slightly higher for the whole frequency range in the expanded computation. The comparative directivity plots of the 3 selected frequencies agree very well (s. Figure 11).

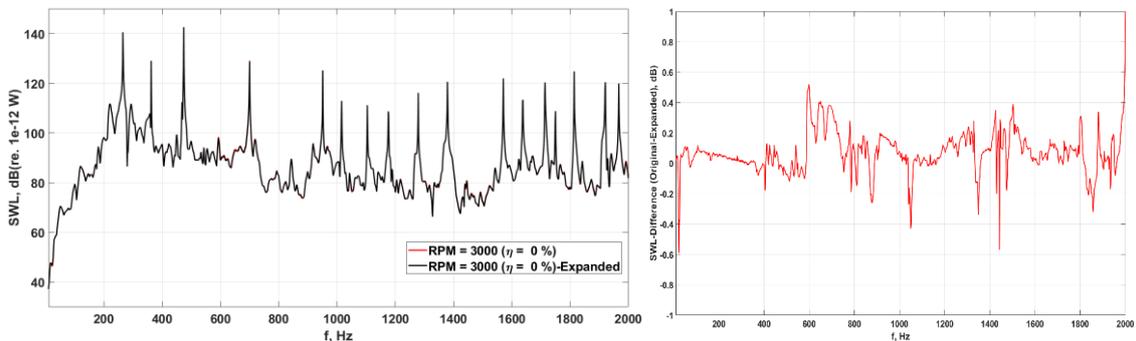


Figure 8: Comparison of ‘measured’ and ‘expanded’ sound power levels (left) and level difference (right)

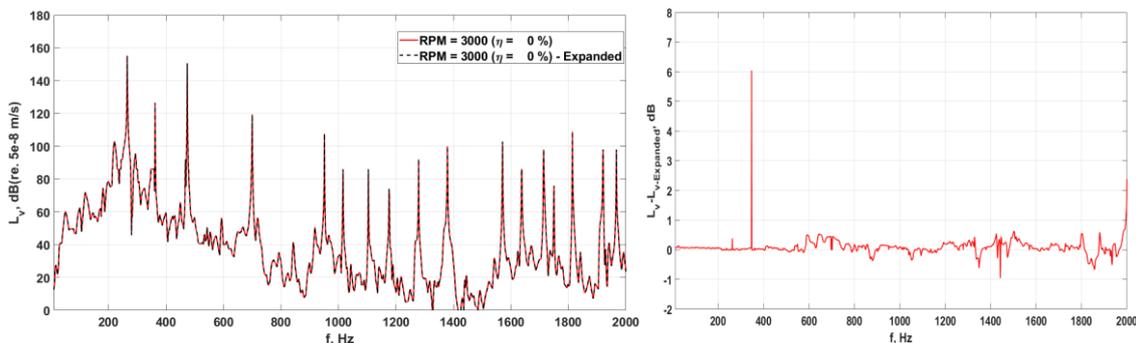


Figure 9: Comparison of ‘measured’ and ‘expanded’ velocity levels (left) and level difference (right)

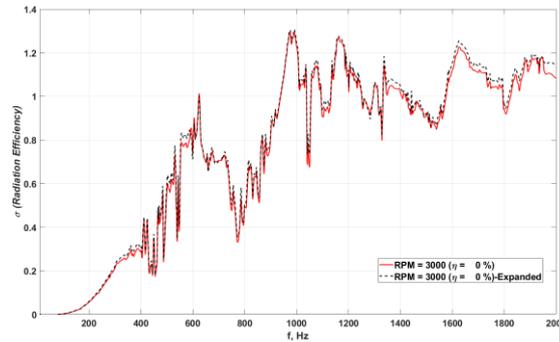


Figure 10: Comparison of 'measured' and 'expanded' radiation efficiency

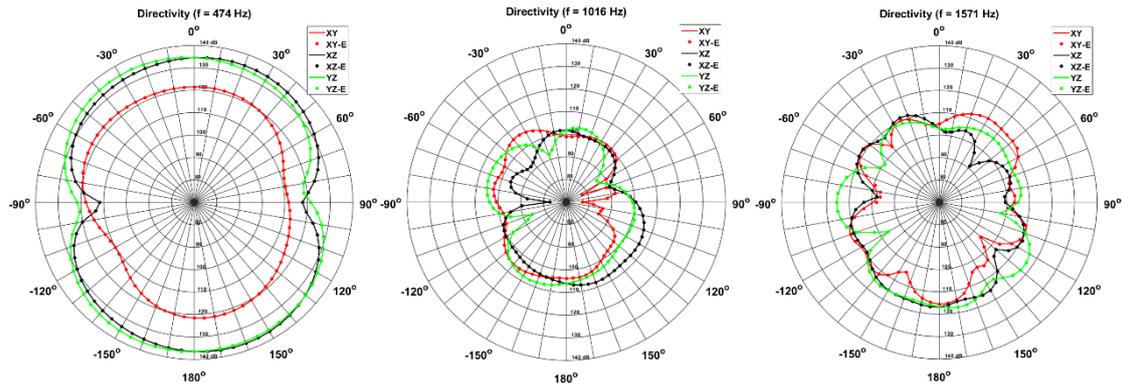


Figure 11: Comparison of 'measured' and 'expanded' directivity

7. CONCLUSIONS

A computer simulation of an operational gear unit was used as a test bench setup for the determination of the acoustic power and directivity. These quantities 'measured' with virtual microphones are considered the 'true measurement results' and therefore the basis for comparison. The aim of this investigation was to determine the emitted airborne sound power and directional characteristics of the gear unit based on a pure vibration measurement with as few vibration sensors as necessary. It was discussed that vibration sensors on their own are in general unable to provide information for a full air-borne acoustic system analysis, that at least an accurate relationship to the structure-borne sound needs to be known. The procedure combines, therefore, an experimental measurement with computational acoustics. The proposed methodology sets two requirements:

1. FE-modal analysis of the gear housing (not an operational simulation, modal damping is not required);
2. Operational measurement.

The chosen procedure is a systems approach: the baseline FEA modal analysis is used to reduce the number of vibration sensors to a minimum, which theoretically still enables a full analysis of all modes of interest. Practical hints and tools were given for some pre-test procedures like the selection of sensor positions and directions. The number of vibration spectra of an operational measurement (ODS) limited due to the reduced number of vibrational computer sensors was expanded with the computational mode shapes of an FEA. The expanded ODS was mapped on an acoustic FE-model and the radiated sound power and directivity was computed. The very good correspondence for the gearbox sound power spectra as well as the directivity plots between 'perfect measurement' and 'the expanded measurement' based on a limited number of vibration sensor surprises. Accurate operational measurement and a valid modal shape analysis

seem to be the only necessity. One can expect complete results from a system-based analysis: this virtual measurement on the computer has delivered very good results. Of course, the question arises whether the procedure described here, which is practically part of the method finding in this study: pre-test vibration sensor selection, ODS-FRF, OMA etc. is really a necessity and decisive for the quality of the results. Three tools are required:

1. FEA tool for modal analysis of the gear housing;
2. Measurement tool providing response complex spectra;
3. Analysis tool for measurement expansion providing the mathematics for the expansion.

8. ACKNOWLEDGMENTS

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