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Vibration absorption in a mistuned bladed disk based on the passive targeted energy transfer

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ABSTRACT

Mistuning may result in the amplification of vibratory amplitude in the blades. In this paper, the inducement of passive nonlinear sink as absorbers for vibration attenuation of a mistuned bladed disk is studied. A single-mode lumped parameter model is developed and the passive targeted energy transfer phenomena occurring between the blades and the nonlinear absorbers is simulated. The influence of mistuning on the blade response is examined by using the Monte Carlo simulations. Moreover, comparative studies of the performance of passive nonlinear sink and undamped linear vibration absorbers also performed. The efficacy of using such passive nonlinear absorbers for the mistuned bladed disks is discussed.

Keywords: Vibration absorber, Nonlinear sink, Mistuned bladed disk, Targeted energy transfer
I-INCE Classification of Subject Number: 46

1. INTRODUCTION

Bladed disks are used widely in many important engineering systems such as turbine generators. Mistuning effects due to manufacturing tolerances may result in the amplification of vibratory amplitude known as localization in the blades, which may be responsible for high-cycle-fatigue (HCF) failure in turbomachinery [1]. Thus, there is an urgent need for efficient control of the dynamic responses of the blades.

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Various ways have been proposed to suppress the vibrations of bladed disks, such as the blade friction damping [2], viscoelastic damping treatment [3], or the piezoelectric shunt damping[4] and so on. As an alternative to the aforementioned options, the nonlinear targeted energy transfer (TET) is also a potential avenue for vibration attenuation of a mistuned bladed disk. TET actually works as an nonlinear absorber, which passively controlled transfers vibrational energy in primary structures to a targeted lightweight component [5,6] by nearly irreversible transfer of broadband energy. Therefore, this paper presents a fundamental study of TET for a mistuned bladed disk to address the possibility of a purely passive integration of the nonlinear vibration absorbers for turbomachinery structures.

2. PROBLEM FORMULATION

2.1 Mistuned bladed disk modeling

Consider a bladed disk system which is represented by a rotationally cyclic chain of spring-mass oscillators as shown in Fig. 1. Each sector is modeled as a one-degree-of-freedom system and adjacent sectors are coupled by linear springs. The mass and stiffness of the *i*-th blade are denoted by m_b^i and k_b^i, while the mechanical coupling between adjacent sectors due to the disk flexibility is represented by k_c. A cubic nonlinear absorber of stiffness k_{a}^i is attached to the *i*-th blade.

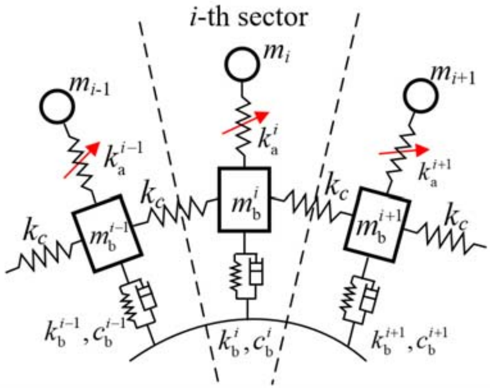

Fig.1 Lumped parameter bladed disk model with nonlinear vibration absorbers

Geometric mistuning is considered and modeled by perturbations in stiffness k_b^i:

$$k_b^i = k_0 + \delta k_b^i = k_0(1+\delta_k^i \xi_k^i), \quad i = 1,\ 2,\ ...,\ N\,, \tag{1}$$

where k_0 is the mean stiffness, ξ_k^i is a random variable with standard normal deviate, δ_k^i is the variation coefficients with regards to the *i*-th blade. Consequentially, the variances of the system responses should also be considered to be random variables. Accordingly, the stochastic governing equations for the *i*-th elementary sector can be written as follows:

$$m_i \ddot{x}_i(\boldsymbol{\xi}_k, t) + k_a^i(\xi_k^i)\left[x_i(\boldsymbol{\xi}_k, t) - y_i(\boldsymbol{\xi}_k, t)\right]^3 = 0\,. \tag{2}$$

$$\begin{aligned} & m_b^i \ddot{y}_i(\boldsymbol{\xi}_k, t) + c_b^i \dot{y}_i(\boldsymbol{\xi}_k, t) + \left[k_b^i(\xi_k^i) + 2k_c\right] y_i(\boldsymbol{\xi}_k, t) \\ & + k_a^i(\xi_k^i)\left[y_i(\boldsymbol{\xi}_k, t) - x_i(\boldsymbol{\xi}_k, t)\right]^3 - k_c\left[y_{i-1}(\boldsymbol{\xi}_k, t) + y_{i+1}(\boldsymbol{\xi}_k, t)\right] = F_i \end{aligned}, \tag{3}$$

where $x_i(\Xi, t)$ and $y_i(\Xi, t)$ are stochastic displacements of the absorber mass and the *i*-th blade mass, respectively, and F_i is the external excitation force of engine order type acting on the *i*-th blade. $\boldsymbol{\xi}_k$ is the N dimension random vector, $\boldsymbol{\xi}_k = \left[\xi_k^1, \xi_k^2, \cdots, \xi_k^N\right]$. For the *r*-th engine-order excitation, F_i can be represented as:

$$F_i(t) = F_0 \mathrm{e}^{\mathrm{j}(i-1)\psi_r + \mathrm{j}\omega t}, \quad r = 1,\ 2,\ ...,\ N, \tag{4}$$

where $\sqrt{\mathrm{j}} = -1$, $\psi_r = 2\pi r/N$, F_0 and ω are amplitude and frequency of the excitation force.

By assembling Eqs. (2)-(3) for the whole system, one can easily obtain the following matrix equations:

$$\mathbf{M}\ddot{\mathbf{d}}(\Xi,\ t) + \mathbf{C}\dot{\mathbf{d}}(\Xi,\ t) + \mathbf{K}(\xi_k, \mathbf{d}(\Xi,\ t)) = \overline{\mathbf{F}}\ . \tag{5}$$

where $\mathbf{d}(\Xi)$ denotes the generalized cordinate vectors of the system, $\overline{\mathbf{F}}$ is the generalized force vector, $\mathbf{M}$ and $\mathbf{C}$ are the deterministic mass and damping matrices, $\mathbf{K}(\boldsymbol{\xi}_k)$ is the random stiffness matrix through which the uncertain geometric mistuning are captured to the structural responses.

2.2 Incremental harmonic balance method

The incremental harmonic balance method is employed here to solve the nonlinear equations in Eq.(5) and only the periodic solutions are concerned with, so the responses can be expanded in the following truncated real-valued Fourier series,

$$\mathbf{d}(t) = \Re\left(\sum_{k=0}^{N_k} Q_k \mathrm{e}^{\mathrm{i}k\Omega t}\right). \tag{6}$$

The substitution of Eq.(6) into Eq.(5) yields the Fourier expansion as

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}(\mathbf{q}) - \mathbf{F}(t) \\ &= \Re\left(\sum_{k=0}^{\infty}\left\{\left[-(k\Omega)^2\mathbf{M} + \mathrm{i}k\Omega\mathbf{C}\right]Q_k + \mathbf{K}_{nl,k} - \mathbf{F}_{ex,k}\right\}\mathrm{e}^{\mathrm{i}k\Omega t}\right) = \Re\left(\sum_{k=0}^{\infty}\mathbf{R}_k \mathrm{e}^{\mathrm{i}k\Omega t}\right) = \mathbf{0} \end{aligned} \tag{7}$$

which gives a set of governing algebraic equations as follows:

$$\mathbf{R}_k = \left[-(k\Omega)^2\mathbf{M} + \mathrm{i}k\Omega\mathbf{C} + \mathbf{K}\right]Q_k + \mathbf{K}_{nl,k}(Q_0, Q_1, \cdots Q_H) - \mathbf{F}_{ex,k} = 0\,. \tag{8}$$

The alternating frequency/time-domain(AFT) technique[2] is then utilized to deal with the vector $\mathbf{K}_{\mathrm{nl,k}}$, which finally provides the estimation of Q_k and then the response $\mathbf{d}$.

2.2 Stochastic responses of the system

The stochastic calculations are carried out through the Monte-Carlo simulations (MCS) by considering N_{MC} samples of the random vector ξ_k. The estimation of the first and second order statistical moments of the responses can be cnsequently given as:

$$\begin{cases} \overline{\mathbf{d}}(\omega_i) = \dfrac{1}{N_{MC}}\displaystyle\sum_{i=1}^{N_{MC}} \mathbf{d}_i(\omega_i) \\ \boldsymbol{\sigma}(\omega_i) = \sqrt{\dfrac{1}{N_{MC}}\displaystyle\sum_{i=1}^{N_{MC}}\left(\mathbf{d}_i(\omega_i) - \overline{\mathbf{d}}(\omega_i)\right)} \end{cases}. \tag{9}$$

3. NUMERICAL RESULTS

To illustrate the application of the nonlinear absorber in the vibration suppression of the mistuned bladed disk, in this section a practical example with N=10 blades is solved and compared with the case of linear absorber as in Ref.[7]. The system parameters are given as follows: $m_b^i = 0.0114$, $k_b^i = 430\,000$, k_c =4543 and c_t=0.138. The order of the engine excitation considered here is 1, while m_i=0.1 m_b^i and

$$k_a^i = \frac{m_i\left(k_b^i + 4k_c\sin^2(\varphi_r/2)\right)}{m_b^i}\,. \tag{10}$$

A typical 1% mistuning in practice is used in this simulation, which is expressed in terms of the ratio of standard deviation and mean value of the modal frequency. As in [7], the amplitudes of the responses for both the blades and the absorber masses are computed between $0.95\omega_r$ and $1.05\omega_r$, where ω_r is the design frequency for the linear vibration absorbers. The maximum amplitude is then computed at each excitation frequency, and the peak maximum amplitude can be subsequently collected by taking maximum over the considered frequency range. Here resonant amplitudes are nondimensionalized with respect to that of the system without vibration absorbers.

Figure 2 shows the peak maximum amplitude for 1000 simulation samples of random mistuning in the linear absorber case. The results are almost the same as that in [7], where the amplitudes for both the absorber masses and blade masses are quite small as compared to the system without vibration absorbers (around 1.8). Moreover, the using of the vibration absorbers has made the responses become not sensitive to mistuning and significantly reduce the localization phenomena.

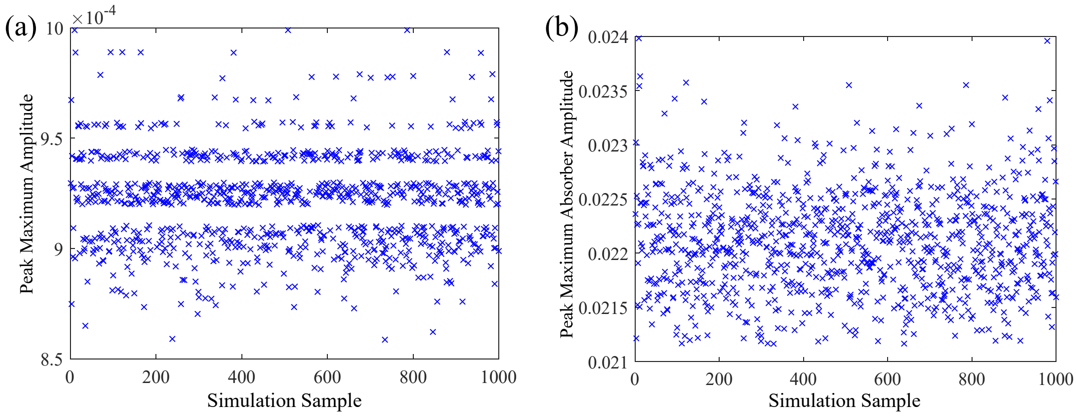

Fig.2 Nondimensional peak maximum amplitude with linear vibration absorbers: (a) the blades, (b) the absorber masses.

The peak maximum amplitude for the 100 simulation samples in the case of the nonlinear absorber is shown in Fig. 3. Further, it is found the peak maximum amplitude of the absorber masses is also very small and keeps in the same order of magnitude as that in Fig. 2. However, it is observed that the peak maximum amplitude of the blades masses become smaller than that in the case of the linear absorbers. Further simulations have also shown that the slight change of the stiffness k_a should not change the peak maximum amplitude too much and absorbers designed based on a specified engine order excitation can also be effective for other engine order excitations because of the essentially nonlinear effect.

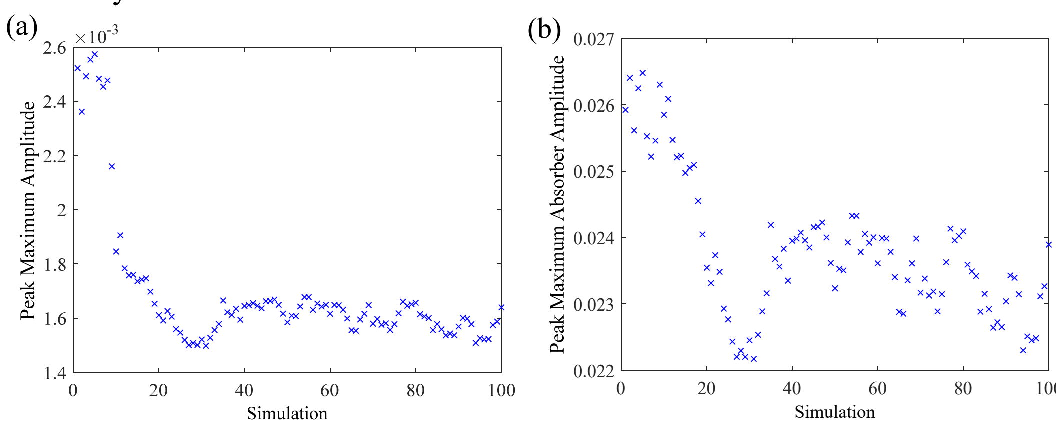

Fig.3 Nondimensional peak maximum amplitude with nonlinear vibration absorbers: (a) the blades, (b) the absorber masses.

4. CONCLUSIONS

In this work, effects of cubic nonlinear vibration absorbers are examined for a lumped parameter bladed disk model with single degree of per sector. The effects of mistuning on the blade response are calculated by using the Monte Carlo simulations. The results reveal that the using of vibration absorbers may let the system responses become less sensitive to the mistuning effect, and the nonlinear absorbers may further reduce the localization phenomena. Further work is underway to explore the mechanism of energy pumping/resonance capture phenomenon, and a robust optimization design to minimize the amplitude of vibration over a wide frequency range should be involved.

5. ACKNOWLEDGEMENTS

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