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Theoretical Analysis on a Nonlinear Modulation of High Speed Ultrasound in Compressible Liquids Containing Many Microbubbles

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ABSTRACT

This paper investigates theoretically an weakly nonlinear propagation of high frequency ultrasonic wave in an initially quiescent liquid containing uniformly many small spherical microbubbles. The incident wave frequency is greater than an eigenfrequency of single bubble oscillations and the phase velocity is greater than the speed of sound in a pure liquid. Remarking that the compressibility of the liquid phase is incorporated. The thermal dissipation effect is discarded. The basic set of equations is composed of the conservation laws of mass and momentum in a two-fluid model, equation of bubble dynamics, equations of state for each phase, and so on. By the use of the method of multiple scales with an appropriate choice of scaling relations composed of three nondimensional parameters in terms of nondimensional wave amplitude, two types of the nonlinear Schröedinger (NLS) equations with some correction terms are derived from the basic set. Then, both the NLS equations govern the competition of nonlinear, dissipation, and dispersion effects. Finally, we discuss the difference between two types of NLS equations derived here.

Keywords: Bubbly liquid, Precursor, Pressure wave, Nonlinear Schrödinger equation
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1. INTRODUCTION

One of the most important properties of pressure wave propagation in bubbly liquids is a dispersion effect of waves [1]. Although the wave frequency is independent of the wavelength in single phase fluids, the wave frequency is dependent on the wavelength in bubbly liquids due to the wave dispersion.

Figure 1 portrays a conceptual diagram of the linear dispersion relation in bubbly liquids, which shows the apparance of two branches, i.e., Slow mode and Fast mode [3,4]. Slow mode and Fast mode correspond to the phase velocity is always lower and higher than the speed of sound in a pure water, respectively [2–4]. Fast mode appears only in the case that the compressibility of the liquid phase is incorporated. Although Slow mode was found about 50 years ago and has a long history of theoretical studies [1–18] , few studies (see, e.g., Refs. [2–4]) have been carried out for Fast mode since the liquid compressibility has long been neglected in most studies. The wave in Fast mode is sometimes called as a precursor, was not well known until recent years. On the other hand, in the experimental studies, waves propagating at a speed close to the speed of sound in a pure water ahead of shock waves, were observed [19]. Since the amplitude of observed waves is very small, further progresses may be supressed. Therefore, a theoretical prediction for Fast mode has long been significantly desired. Since amplitude of such a wave is small but finite as a real phenomena, a weakly nonlinear analysis (i.e., neither linear nor strongly nonlinear analyses) [20] is appropriate to the prediction.

The aim of this paper is a theoretical prediction for a fast propagation of quasi-monochromatic waves at a high-frequency exceeding the eigenfrequency of single bubble oscillations in compressible bubbly liquids. As a result, two types of nonlinear wave equations are derived and resultant nonlinear wave equations describe waves propagating at a high phase velocity by taking the effect of liquid compressibility in consideration.

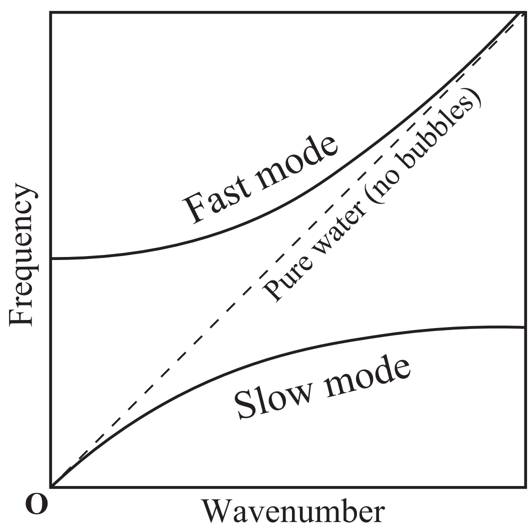

Figure 1: A conceptual diagram of linear dispersion relation for pressure disturbance in compressible bubbly liquids [2–4].

2. PROBLEM STATEMENT

We tackle a theoretical investigation for two types of fast propagations of plane progressive pressure waves in an initially quiescent compressible liquids uniformly containing many spherical microbubbles on the basis of the derivation of two types of nonlinear wave equations describing long-range propagation of quasi-monochromatic waves in bubbly liquids.

Main assumptions are summarized as follows: (i) The wave frequency is larger than an eigenfrequency of single bubble oscillations; (ii) The compressibility of the liquid phase is incorporated; (iii) The phase velocity is larger than the speed of sound in a pure liquid; (iv) The bubble does not coalesce, break up, extinct, and appear; (v) The viscosity of the liquid phase is taken into account only at the bubble-liquid interface, although that of the gas phase is omitted; (vi) The bulk viscosities of the gas and liquid phases are neglected; (vii) The effect of viscosity in the gas phase, heat conduction in the gas and liquid phases, phase change across the bubble wall, and thermal conductivities of the gas and liquid, are neglected.

3. GOVERNING EQUATIONS

The conservation laws of mass and momentum for gas and liquid phases for bubbly flows based on a two-fluid model [3] are firstly employed:

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) = 0, \tag{1}$$

$$\frac{\partial}{\partial t^*}\left[(1-\alpha)\rho_{\mathrm{L}}^*\right] + \frac{\partial}{\partial x^*}\left[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*\right] = 0, \tag{2}$$

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}\left(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^{*2}\right) + \alpha\frac{\partial p_{\mathrm{G}}^*}{\partial x^*} = F^*, \tag{3}$$

$$\frac{\partial}{\partial t^*}\left[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*\right] + \frac{\partial}{\partial x^*}\left[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^{*2}\right] + (1-\alpha)\frac{\partial p_{\mathrm{L}}^*}{\partial x^*} + P^*\frac{\partial \alpha}{\partial x^*} = -F^*, \tag{4}$$

where t^* is the time, x^* space coordinate, α void fraction, ρ^* density, u^* fluid velocity, p^* pressure, and P^* pressure averaged on the bubble–liquid interface; the subscripts G and L denote volume-averaged variables in gas and liquid phases, respectively; the superscript asterisk denotes dimensional quantity. We then introduce the interfacial momentum transport F^* as the following model of a virtual mass force [4, 21]:

$$F^* = -\beta_1\alpha\rho_{\mathrm{L}}^*\left(\frac{\mathrm{D_G}u_{\mathrm{G}}^*}{\mathrm{D}t^*} - \frac{\mathrm{D_L}u_{\mathrm{L}}^*}{\mathrm{D}t^*}\right) - \beta_2\rho_{\mathrm{L}}^*(u_{\mathrm{G}}^* - u_{\mathrm{L}}^*)\frac{\mathrm{D_G}\alpha}{\mathrm{D}t^*} - \beta_3\alpha(u_{\mathrm{G}}^* - u_{\mathrm{L}}^*)\frac{\mathrm{D_G}\rho_{\mathrm{L}}^*}{\mathrm{D}t^*}, \tag{5}$$

where the values of coefficients $\beta_j\,(j = 1, 2, 3)$ may be set as $1/2$ for a spherical bubble, and $\mathrm{D}/\mathrm{D}t^*$ stands for the Lagrange differential operator:

$$\frac{\mathrm{D_G}}{\mathrm{D}t^*} = \frac{\partial}{\partial t^*} + u_{\mathrm{G}}^*\frac{\partial}{\partial x^*}, \quad \frac{\mathrm{D_L}}{\mathrm{D}t^*} = \frac{\partial}{\partial t^*} + u_{\mathrm{L}}^*\frac{\partial}{\partial x^*}. \tag{6}$$

The Keller–Miksis equation [22] for spherical symmetric oscillations of a reprentative bubble in a compressible liquid is given by

$$\left(1 - \frac{1}{c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)R^*\frac{\mathrm{D_G^2}R^*}{\mathrm{D}t^{*2}} + \frac{3}{2}\left(1 - \frac{1}{3c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)\left(\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)^2$$
$$= \left(1 + \frac{1}{c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)\frac{P^*}{\rho_{\mathrm{L0}}^*} + \frac{R^*}{\rho_{\mathrm{L0}}^* c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}}{\mathrm{D}t^*}(p_{\mathrm{L}}^* + P^*), \tag{7}$$

where $R^*(x^*, t^*)$ is a representative bubble radius and c^*_{L0} is the speed of sound in pure water. The averaged bubble radius R^* is not only a function of time t^* but also a function of space coordinate x^*. Although many previous studies used partial differential operator as a time derivative, we used the Lagrange derivative with respect to gas phase, i.e., $\mathrm{D_G}/\mathrm{D}t$.

Let us close the set of Eqs. (1)–(7) by imposing the following supplementary equations, i.e., the polytropic equation of state of gas and Tait's equation of state of liquid to describe a compressible fluid, the conservation equation of mass inside the bubble, and the balance of normal stresses across the bubble–liquid interface:

$$\begin{aligned} \frac{p^*_{\mathrm{G}}}{p^*_{\mathrm{G0}}} &= \left(\frac{\rho^*_{\mathrm{G}}}{\rho^*_{\mathrm{G0}}}\right)^{\gamma}, \quad p^*_{\mathrm{L}} = p^*_{\mathrm{L0}} + \frac{\rho^*_{\mathrm{L0}} c^{*2}_{\mathrm{L0}}}{n}\left[\left(\frac{\rho^*_{\mathrm{L}}}{\rho^*_{\mathrm{L0}}}\right)^{n} - 1\right], \\ \frac{\rho^*_{\mathrm{G}}}{\rho^*_{\mathrm{G0}}} &= \left(\frac{R^*_0}{R^*}\right)^{3}, \quad p^*_{\mathrm{G}} - (p^*_{\mathrm{L}} + P^*) = \frac{2\sigma^*}{R^*} + \frac{4\mu^*}{R^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}, \end{aligned} \tag{8}$$

where γ is the polytropic exponent, n material constant, σ^* surface tension, and μ^* liquid viscosity; the physical quantities in the initial unperturbed state are signified by the subscript 0, and they are all the constants.

Noting that this study incorporates only the effect of liquid viscosity acting on the bubble–liquid interface since it is not a negligible effect in bubbly liquids. The effect of liquid viscosity appears at far field II.

4. MULTIPLE SCALES ANALYSIS

Based on a relationship of $U^* = L^*\omega^*$ among a typical group velocity U^*, a typical wavelength L^*, and a typical frequency of incident wave ω^*, we determine the sizes of three nondimensional ratios appropriate to the low frequency and high frequency bands of Fast mode, as follows:

$$\left(\frac{U^*}{c^*_{\mathrm{L0}}}, \frac{R^*_0}{L^*}, \frac{\omega^*}{\omega^*_{\mathrm{B}}}\right) \equiv \begin{cases} \left(V\epsilon, \mathit{\Delta}, \mathit{\Omega}\epsilon^{-1/2}\right) & \text{(low frequency)} \\ \left(V, \mathit{\Delta}, \mathit{\Omega}\epsilon^{-1/2}\right) & \text{(high frequency)} \end{cases} \tag{9}$$

where the perturbation ϵ ($\ll 1$) is a typical nondimensional wave amplitude; V, $\mathit{\Delta}$, and $\mathit{\Omega}$ are all the constants of $O(1)$; ω^*_{B} is the natural angular frequency of single bubble oscillations ,

$$\omega^*_{\mathrm{B}} \equiv \sqrt{\frac{3\gamma(p^*_{\mathrm{L0}} + 2\sigma^*/R^*_0) - 2\sigma^*/R^*_0}{\rho^*_{\mathrm{L0}} R^{*2}_0}}. \tag{10}$$

The time t^* and space coordinate x^* are nondimensionalized as $t \equiv t^*/T^*$ and $x \equiv x^*/L^*$, respectively. Then, we define six new independent variables for near field, far field I [i.e., the temporal and spatial scales of $O(1/\epsilon)$], and far field II [i.e., the temporal and spatial scales of $O(1/\epsilon^2)$]:

$$t_0 = t, \quad x_0 = x; \qquad t_1 = \epsilon t, \quad x_1 = \epsilon x; \qquad t_2 = \epsilon^2 t, \quad x_2 = \epsilon^2 x. \tag{11}$$

We then nondimensionalize and expand the dependent variables in power series of ϵ, as follows:

$$\frac{\alpha}{\alpha_0} = 1 + \epsilon\alpha_1 + \epsilon^2\alpha_2 + O(\epsilon^3), \tag{12}$$

$$\frac{u_{\rm G}^*}{U^*} = \epsilon u_{\rm G1} + \epsilon^2 u_{\rm G2} + O(\epsilon^3), \tag{13}$$

$$\frac{u_{\rm L}^*}{U^*} = \epsilon u_{\rm L1} + \epsilon^2 u_{\rm L2} + O(\epsilon^3), \tag{14}$$

$$R^*/R_0^* = 1 + \epsilon R_1 + \epsilon^2 R_2 + O(\epsilon^3), \tag{15}$$

The expansion of the liquid density is divided in two types of expansions:

$$\frac{\rho_{\rm L}^*}{\rho_{\rm L0}^*} = \begin{cases} 1 + \epsilon^2\rho_{\rm L1} + \epsilon^3\rho_{\rm L2} + O(\epsilon^4), & \text{(low frequency)}, \\ 1 + \epsilon\rho_{\rm L1} + \epsilon^2\rho_{\rm L2} + O(\epsilon^3), & \text{(high frequency)}, \end{cases} \tag{16}$$

and these exponents of ϵ are uniquiely determined without any ambiguity [23, 24].

5. RESULT: EACH ORDER OF APPROXIMATIONS

Equating the coefficients of like powers of ϵ in Eqs. (1)–(4) and (7), approximate equations for each order of approximation are derived as two types of resultant equations, i.e., waves in low and high frequencies. Noting that the detailed derivation procedure for the low frequency case has been provided in Ref. [25]

(i) Leading order of approximation—— linearized partial differential equations for the first-order perturbation of the bubble radius, R_1, are derived:

$$\frac{\partial^2 R_1}{\partial t_0^2} - \frac{(1-\alpha_0+\beta_1)\gamma p_{\rm G0}}{\beta_1(1-\alpha_0)V^2}\frac{\partial^2 R_1}{\partial x_0^2} - \frac{\varDelta^2}{3\alpha_0}\frac{\partial^4 R_1}{\partial x_0^2 \partial t_0^2} = 0, \quad \text{(low frequency)}, \tag{17}$$

$$\frac{\partial^2 R_1}{\partial t_0^2} - \frac{(1-\alpha_0+\beta_1)\gamma p_{\rm G0}}{\beta_1(1-\alpha_0)}\frac{\partial^2 R_1}{\partial x_0^2} - \frac{\varDelta^2}{3\alpha_0}\frac{\partial^4 R_1}{\partial x_0^2 \partial t_0^2} + \frac{\varDelta^2 V^2(1-\alpha_0)}{3\alpha_0}\frac{\partial^4 R_1}{\partial t_0^4} = 0, \quad \text{(high frequency)}. \tag{18}$$

Let us proceed the derivation of the approximation equations for far field I [i.e., $O(\epsilon^2)$] and far field II [i.e., $O(\epsilon^3)$]. By using the solvability conditions of these inhomogeneous equations, we have the following approximate equations.

(ii) Second order of approximation—— approximate equations for far field I are

$$\frac{\partial A}{\partial t_1} + v_{\rm g}\frac{\partial A}{\partial x_1} + {\rm i}\eta A = 0, \quad \text{(low frequency)}, \tag{19}$$

$$\frac{\partial A}{\partial t_1} + v_{\rm g}\frac{\partial A}{\partial x_1} + {\rm i}\lambda A = 0, \quad \text{(high frequency)}, \tag{20}$$

where $v_{\rm g}$ is the group velocity, η is the real constant [25], and λ is the complex constant. Then, the complex amplitude as the dependent variable, A, is defined via R_1:

$$R_1 = A(t_1, t_2, x_1, x_2){\rm e}^{{\rm i}\theta} + \text{c.c.}, \quad \theta = kx_0 - \varOmega(k)t_0, \tag{21}$$

where $k \equiv k^*L^*$ is the nondimensional wavenumber (k^* is the wavenumber), and θ is the phase function. The slow variation of the carrier wave ${\rm e}^{{\rm i}\theta}$ is described by the envelope wave A, that is, a slowly modulated wave packet A is induced by the weakly nonlinear propagation of quasi-monochromatic carrier wavetrain ${\rm e}^{{\rm i}\theta}$.

(iii) Third order of approximation—— approximate equations for far field II are

$$\mathrm{i}\left(\frac{\partial A}{\partial t_2}+v_{\mathrm{g}}\frac{\partial A}{\partial x_2}\right)+\frac{q}{2}\frac{\partial^2 A}{\partial x_1^2}+C_1|A|^2A+\mathrm{i}C_2A+C_3A+\mathrm{i}C_4\frac{\partial A}{\partial x_1}=0, \quad \text{(low frequency)}, \tag{22}$$

$$\mathrm{i}\left(\frac{\partial A}{\partial t_2}+v_{\mathrm{g}}\frac{\partial A}{\partial x_2}\right)+\frac{q}{2}\frac{\partial^2 A}{\partial x_1^2}+D_1|A|^2A+D_2A+D_3\frac{\partial A}{\partial x_1}=0, \quad \text{(high frequency)}, \tag{23}$$

where $q\,(=\mathrm{d}v_{\mathrm{g}}/\mathrm{d}k)$ is the derivative of group velocity with respect to the wavenumber, $C_j(j=1,2,3,4)$ are the real constants [25], and $D_j(j=1,2,3)$ are the complex constants.

6. RESULT: NONLINEAR SCHRÖDINGER EQUATIONS

Combining the results for near field, far field I, and far field II, we have two types of nonlinear Schrödinger equations:

$$\begin{aligned}&\mathrm{i}\frac{\partial A}{\partial \tau}+\frac{q}{2}\frac{\partial^2 A}{\partial \xi^2}+C_1|A|^2A+\mathrm{i}C_2A=0,\\&\quad\tau=\epsilon^2\left(1-\frac{C_3}{W}\right)t,\quad \xi=\epsilon\left[x-\left(v_{\mathrm{g}}+\frac{\eta}{K}+\epsilon C_4\right)t\right],\quad \text{(low frequency)},\end{aligned} \tag{24}$$

$$\begin{aligned}&\mathrm{i}\frac{\partial A}{\partial \tau}+\frac{q}{2}\frac{\partial^2 A}{\partial \xi^2}+D_1|A|^2A+D_2A+D_3\frac{\partial A}{\partial \xi}=0,\\&\quad\tau=\epsilon^2 t,\quad \xi=\epsilon(x-v_{\mathrm{g}}t),\quad \text{(high frequency)}.\end{aligned} \tag{25}$$

In Eq. (24), W and K are the nondimensional frequency and wavenumber of the envelope wave, respectively. Equations (24)(25) are two types of nonlinear Schrödinger (NLS) equations containing some correction terms. In Eq. (24), the fourth term in the left-hand side is an attenuation term because of $C_2 \geq 0$, and the wave attenuation is owing to the effects of liquid viscosity and liquid compressibility. Therefore, the dispersion, nonlinearity, and dissipation of envelope wave appear and compete with each other in far field.

7. SUMMARY

We theoretically examine two types of fast propagations of plane progressive pressure waves in uniform bubbly liquids. By using the method of multiple scales with an appropriate choice of set of scaling relations of nondimensional parameters, two types of the NLS equations with some correction terms, Eqs. (24)(25), can be derived from the governing equations, which describe the long-range wave propagation with the dissipation and dispersion effects. The resulutant NLS equations are different compared with the previously derived NLS equations [23, 24] for Slow mode where the phase velocity is always smaller than the speed of sound in pure water. The coefficients for NLS equations contains the parameter V owing to the effect of liquid compressibility. The detailed physics such as a waveform derived from Eq. (24) and a physico-mathematical explanation of Eq. (25) will be provided in a presentation.

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