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Thermal and Viscous Damping of Propagation Process of Weakly Nonlinear Waves in Bubbly Liquids

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ABSTRACT

This study theoretically examines an weakly nonlinear pressure disturbance in an initially quiescent liquid uniformly containing many spherical microbubbles, especially focusing on an effect of the liquid viscosity and the thermal conductivity on the wave propagation process. The use of method of multiple scales (e.g., Jeffrey & Kawahara, 1982) and of parameter scaling appropriate to a low frequency compared with the eigenfrequency of single bubble oscillations and a long wavelength compared with the bubble radius (Kanagawa et al., 2011) results a systematic derivation of a far field equation (i.e., nonlinear wave equation). The main results are summarized as follows: (i) The Korteweg–de Vries–Burgers equation incorporating the liquid viscosity and thermal conductivity was derived; (ii) The incorporation of energy equation affects the nonlinear, dispersion, and dissipation terms; (iii) The liquid viscosity and the thermal conductivity lead to change considerably the explicit form of coefficient of dissipation term.

Keywords: Bubble dynamics, Pressure wave, Dissipation, KdV equation
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1. INTRODUCTION

The propagation of pressure waves (or acoustic waves) in water containing microbubbles observed in various fields of basic physics and engineering application [1–16]. While the formation of shock waves in bubbly liquids negatively affects a fluid machinery, this is expected to be applied to various medical applications such as a

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shock wave lithotripsy and tumor treatment. In order to develop such an application, an appropriate formulation of the dissipation effect of waves in bubbly liquids is significantly important.

Although our previous studies [17, 18] have derived the Korteweg–de Vries–Burgers (KdVB) equation for a low frequency long wave in bubbly liquids, the dissipation effect due to the liquid viscosity and the thermal conductivity, have been ignored. The purpose of this paper is to re-derive the KdVB equation by using governing equations incorporating the liquid viscosity and thermal conductivity. Thus, we can clarify the effects of the liquid viscosity and thermal conductivity on the propagation process of pressure waves.

2. PROBLEM STATEMENT

We shall examine plane progressive waves in an initially quiescent liquid uniformly containing a number of small spherical gas bubbles. The incident frequency of waves is considerably lower than the eigenfrequency of bubble oscillations, and the wavelength is considerably longer than the bubble radius. The liquid phase is assumed as the Newtonian fluid and the heat flux obeys Fourier's law.

For simplicity, the viscosity of the gas phase, the phase change and mass transport across the bubble–liquid interface, are ignored. The bubbles do not coalesce, break up, disappear, and appear.

3. GOVERNING EQUATIONS

For one-dimensional case, the conservation equations of mass, momentum, and energy in bubbly liquids are written as follows [19]:

$$\frac{\partial \rho^*}{\partial t^*} + \frac{\partial \rho^* u^*}{\partial x^*} = 0, \tag{1}$$

$$\frac{\partial \rho^* u^*}{\partial t^*} + \frac{\partial \rho^* u^{*2}}{\partial x^*} + \frac{\partial p_{\mathrm{L}}^*}{\partial x^*} - \frac{4}{3}\mu^* \frac{\partial^2 u^*}{\partial x^{*2}} = 0, \tag{2}$$

$$\frac{\partial \rho^* h^*}{\partial t^*} + \frac{\partial \rho^* h^* u^*}{\partial x^*} - \frac{\partial p_{\mathrm{L}}^*}{\partial t^*} - u^* \frac{\partial p_{\mathrm{L}}^*}{\partial x^*} - \frac{4}{3}\mu^* \left(\frac{\partial u^*}{\partial x^*}\right)^2 - \lambda^* \frac{\partial^2 T^*}{\partial x^{*2}} = 0, \tag{3}$$

where t^* is the time, x^* space coordinate, ρ^* density, u^* fluid velocity, p^* pressure, h^* enthalpy per unit mass, T^* temperature, μ^* viscosity (constant), and λ^* thermal conductivity (constant); the subscript L denotes the volume-averaged variable in the liquid phase; asterisk * denotes the dimensional quantity. Noting that our previous studies [17, 18] neglected the the viscosity in Eq. (2) and the energy equation (3).

The volume averaged density of the mixture, ρ^*, is defined by

$$\rho^* \equiv (1 - \alpha)\rho_{\mathrm{L}}^*, \tag{4}$$

where α is the void fraction, and the volume averaged density of the gas is neglected. Now, α is connected by the number density of bubbles, N^*:

$$\alpha = \frac{4}{3}\pi R^{*3} N^*, \tag{5}$$

$$\frac{\partial N^*}{\partial t^*} + \frac{\partial N^* u^*}{\partial x^*} = 0, \tag{6}$$

where R^* is a representative bubble radius. Equation (5) defines α and Eq. (6) corresponds to the conservation law of N.

Substituting Eqs. (5) and (15) below into Eq. (6), and then substituting Eq. (4) into Eqs. (1)–(3), we have

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}(\alpha\rho_{\mathrm{G}}^* u^*) = 0, \tag{7}$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^*] + \frac{\partial}{\partial x^*}[(1-\alpha)\rho_{\mathrm{L}}^* u^*] = 0, \tag{8}$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^* u^*] + \frac{\partial}{\partial x^*}\left[(1-\alpha)\rho_{\mathrm{L}}^* u^{*2}\right] + \frac{\partial p_{\mathrm{L}}^*}{\partial x^*} - \frac{4}{3}\mu^*\frac{\partial^2 u^*}{\partial x^{*2}} = 0, \tag{9}$$

$$\rho^* c_{\mathrm{P}}^* \frac{\mathrm{D}T^*}{\mathrm{D}t^*} - \beta^* T^* \frac{\mathrm{D}p_{\mathrm{L}}^*}{\mathrm{D}t^*} - \frac{4}{3}\mu^*\left(\frac{\partial u^*}{\partial x^*}\right)^2 - \lambda^*\frac{\partial^2 T^*}{\partial x^{*2}} = 0, \tag{10}$$

where c_{P}^* is the isobatric specific heat, β^* is the volume expansion coefficient, and the subscript G denotes the volume-averaged variable in the gas phase.

The Keller equation [20] for spherical symmetric oscillations of a representative bubble in a compressible liquid is given by

$$\begin{aligned}\left(1 - \frac{1}{c_{\mathrm{L0}}^*}\frac{\mathrm{D}R^*}{\mathrm{D}t^*}\right) R^* \frac{\mathrm{D}^2 R^*}{\mathrm{D}t^{*2}} + \frac{3}{2}\left(1 - \frac{1}{3c_{\mathrm{L0}}^*}\frac{\mathrm{D}R^*}{\mathrm{D}t^*}\right)\left(\frac{\mathrm{D}R^*}{\mathrm{D}t^*}\right)^2 \\ = \left(1 + \frac{1}{c_{\mathrm{L0}}^*}\frac{\mathrm{D}R^*}{\mathrm{D}t^*}\right)\frac{P^*}{\rho_{\mathrm{L0}}^*} + \frac{R^*}{\rho_{\mathrm{L0}}^* c_{\mathrm{L0}}^*}\frac{\mathrm{D}}{\mathrm{D}t^*}(p_{\mathrm{L}}^* + P^*),\end{aligned} \tag{11}$$

where c_{L0}^* is the speed of sound in the initial unperturbed pure water, the subscript 0 denotes the initial unperturbed state, and the material differential operator $\mathrm{D}/\mathrm{D}t^*$ stands for

$$\frac{\mathrm{D}}{\mathrm{D}t^*} = \frac{\partial}{\partial t^*} + u^*\frac{\partial}{\partial x^*}. \tag{12}$$

The system of Eqs. (7)–(11) is closed by the following equations:

(i) Tait's equation of state for liquid phase,

$$p_{\mathrm{L}}^* = p_{\mathrm{L0}}^* + \frac{\rho_{\mathrm{L0}}^* c_{\mathrm{L0}}^{*2}}{n}\left[\left(\frac{\rho_{\mathrm{L}}^*}{\rho_{\mathrm{L0}}^*}\right)^n - 1\right], \tag{13}$$

where n is the material constant.

(ii) Equation of state for ideal gas,

$$\frac{p_{\mathrm{G}}^*}{p_{\mathrm{G0}}^*} = \frac{\rho_{\mathrm{G}}^*}{\rho_{\mathrm{G0}}^*}\frac{T^*}{T_0^*}. \tag{14}$$

We assume that the temperature of gas phase is equivalent to the temperature of the bubbly liquids, i.e., T^*.

(iii) The conservation equation of mass inside the bubble,

$$\frac{\rho_{\mathrm{G}}^*}{\rho_{\mathrm{G0}}^*} = \left(\frac{R_0^*}{R^*}\right)^3. \tag{15}$$

(iv) The balance of normal stresses across the bubble–liquid interface,

$$p_{\mathrm{G}}^{*}-(p_{\mathrm{L}}^{*}+P^{*})=\frac{2\sigma^{*}}{R^{*}}+\frac{4\mu^{*}}{R^{*}}\frac{\mathrm{D}R^{*}}{\mathrm{D}t^{*}}, \tag{16}$$

where σ^* is the surface tension. Remarking that our previous studies [17, 18] have considered the second term in the right-hand of Eq. (16), and have neglected the viscosity in Eqs. (9) and (10) and the thermal conductivity in Eq. (10).

4. MULTIPLE SCALES ANALYSIS

In a problem of the so-called weakly nonlinear waves, where a typical nondimensional amplitude of waves, ϵ, is finite but sufficiently small compared with unity. Various scales of temporal and spatial variations produced by the weak nonlinearity can be incorporated systematically by introducing multiple scales (ϵt, ϵx; $\epsilon^2 t$, $\epsilon^2 x$, ...) via a small parameter ϵ ($\ll 1$) [21].

To do so, the time t^* and the space coordinate x^* are firstly nondimensionalized as $t=\omega^* t^*$ and $x=x^*/L^*$, respectively, where ω^* is the typical angular frequency of incident wave, and L^* is the typical wavelength. Then, new independent variables based on ϵ is defined for near field [i.e., the temporal and spatial scales of $O(1)$] and far field [i.e., the temporal and spatial scales of $O(1/\epsilon)$]:

$$t_0=t, \quad x_0=x; \quad t_1=\epsilon t, \quad x_1=\epsilon x. \tag{17}$$

Dependent variables are nondimensionalized and expanded in power of ϵ:

$$\frac{R^*}{R_0^*}-1=\epsilon R_1+\epsilon^2 R_2+O(\epsilon^3), \tag{18}$$

$$\frac{\alpha}{\alpha_0}-1=\epsilon\alpha_1+\epsilon^2\alpha_2+O(\epsilon^3), \tag{19}$$

$$\frac{T^*}{T_0^*}-1=\epsilon T_1+\epsilon^2 T_2+O(\epsilon^3), \tag{20}$$

$$\frac{u^*}{U^*}=\epsilon u_1+\epsilon^2 u_2+O(\epsilon^3), \tag{21}$$

where U^* is typical phase velocity. Then, the expansion of the liquid density is given by

$$\frac{\rho_{\mathrm{L}}^*}{\rho_{\mathrm{L0}}^*}=1+\epsilon^2\rho_{\mathrm{L1}}+\epsilon^3\rho_{\mathrm{L2}}+O(\epsilon^4), \tag{22}$$

which was determined from Eq. (13) [17, 18]. Furthermore, the pressures are nondimensionalized as

$$p_{\mathrm{L}}=\frac{p_{\mathrm{L}}^*}{\rho_{\mathrm{L0}}^* U^{*2}}, \quad p_{\mathrm{L0}}=\frac{p_{\mathrm{L0}}^*}{\rho_{\mathrm{L0}}^* U^{*2}}, \quad p_{\mathrm{G0}}=\frac{p_{\mathrm{G0}}^*}{\rho_{\mathrm{L0}}^* U^{*2}}, \tag{23}$$

where p_{L0} and p_{G0} are the constants of $O(1)$, and p_{L} is expanded as

$$p_{\mathrm{L}}=p_{\mathrm{L0}}+\epsilon p_{\mathrm{L1}}+\epsilon^2 p_{\mathrm{L2}}+O(\epsilon^3). \tag{24}$$

Subsequently, we determine the sizes of nondimensional parameters appearing in nondimensionalized conservation equations (9) and (10):

$$\left(\frac{\mu^*}{\rho_{\mathrm{L0}}^* U^* L^*}, \frac{\lambda^*}{\rho_{\mathrm{L0}}^* U^* L^* c_{\mathrm{P}}^*}, \frac{\beta^* U^{*2}}{c_{\mathrm{P}}^*}, \frac{U^{*2}}{c_{\mathrm{P}}^* T_0^*}\right)=(\mu\epsilon, \lambda\epsilon, \delta, \eta), \tag{25}$$

where μ, λ, δ, and η are the constants of $O(1)$. Furthermore, there exists a relationship of $U^* = L^*\omega^*$ among U^*, L^*, and ω^*, and we should determine the sizes of three nondimensional parameters, as follows [17, 18]:

$$\left(\frac{U^*}{c^*_{\mathrm{L0}}}, \frac{R^*_0}{L^*}, \frac{\omega^*}{\omega^*_{\mathrm{B}}}\right) = (V\sqrt{\epsilon}, \mathit{\Delta}\sqrt{\epsilon}, \mathit{\Omega}\sqrt{\epsilon}), \tag{26}$$

where V, $\mathit{\Delta}$, and $\mathit{\Omega}$ are the constants of $O(1)$, and ω^*_{B} is the eigenfrequency of single bubble oscillations given by

$$\omega^*_{\mathrm{B}} \equiv \sqrt{\frac{3\gamma(p^*_{\mathrm{L0}} + 2\sigma^*/R^*_0) - 2\sigma^*/R^*_0}{\rho^*_{\mathrm{L0}}R^{*2}_0}}. \tag{27}$$

5. RESULT: LEADING ORDER OF APPROXIMATION

Equating the coefficients of like powers of ϵ in the governing equations (7)–(11), the following set of linearized equations as the first-order equations is derived:

$$\frac{\partial \alpha_1}{\partial t_0} - 3\frac{\partial R_1}{\partial t_0} + \frac{\partial u_1}{\partial x_0} = 0, \tag{28}$$

$$\alpha_0\frac{\partial \alpha_1}{\partial t_0} - (1-\alpha_0)\frac{\partial u_1}{\partial x_0} = 0, \tag{29}$$

$$(1-\alpha_0)\frac{\partial u_1}{\partial t_0} + \frac{\partial p_{\mathrm{L1}}}{\partial x_0} = 0, \tag{30}$$

$$(1-\alpha_0)\frac{\partial T_1}{\partial t_0} - \delta\frac{\partial p_{\mathrm{L1}}}{\partial t_0} = 0, \tag{31}$$

$$-\frac{\mathit{\Delta}^2}{\mathit{\Omega}^2}R_1 - p_{\mathrm{L1}} + p_{\mathrm{G0}}T_1 + 3(\gamma-1)p_{\mathrm{G0}}R_1 = 0. \tag{32}$$

By combining these equations into a single equation, the linear wave equation for the first-order perturbation of the bubble radius, R_1, is derived:

$$\frac{\partial^2 R_1}{\partial t_0^2} - v_{\mathrm{p}}^2\frac{\partial^2 R_1}{\partial x_0^2} = 0, \tag{33}$$

where the phase velocity v_{p} as a constant coefficient is given by

$$v_{\mathrm{p}} = \sqrt{\frac{\mathit{\Delta}^2/\mathit{\Omega}^2 - 3(\gamma-1)p_{\mathrm{G0}}}{3\alpha_0(1-\alpha_0-\delta p_{\mathrm{G0}})}}. \tag{34}$$

Remarking that the appearance of the terms, $-3(\gamma-1)p_{\mathrm{G0}}$ and $-\delta p_{\mathrm{G0}}$, is the essential difference compared with our previous studies [17, 18]. These terms, i.e., δ and γ, represents thermal effects and affect the phase velocity v_{p}.

Now, choosing U^* as

$$U^* = \sqrt{\frac{R^{*2}_0\omega^{2*}_{\mathrm{B}} - 3(\gamma-1)p^*_{\mathrm{G0}}/\rho^*_{\mathrm{L0}}}{3\alpha_0\left[1-\alpha_0-\beta^*p^*_{\mathrm{G0}}/(\rho^*_{\mathrm{L0}}c^*_{\mathrm{P}})\right]}}, \tag{35}$$

gives $v_{\mathrm{p}} \equiv 1$.

From now on, the right-running wave in the leading order of approximation is focused, and a phase function φ_0 is then introduced as

$$\varphi_0 = x_0 - t_0. \tag{36}$$

Then, Eq. (33) reduces to

$$\frac{\partial f}{\partial t_0} + \frac{\partial f}{\partial x_0} = 0, \tag{37}$$

for $f \equiv R_1(\varphi_0)$.

6. RESULT: SECOND ORDER OF APPROXIMATION

Let us proceed the second order of approximation. The following single inhomogeneous wave equation is obtained:

$$\frac{\partial^2 R_2}{\partial t_0^2} - \frac{\partial^2 R_2}{\partial x_0^2} = K(f; \varphi_0, t_1, x_1). \tag{38}$$

From the solvability condition [17, 21] of Eq. (38), we finally obtain the KdVB equation:

$$\frac{\partial f}{\partial \tau} + \Pi_1 f \frac{\partial f}{\partial \xi} + \Pi_2 \frac{\partial^2 f}{\partial \xi^2} + \Pi_3 \frac{\partial^3 f}{\partial \xi^3} = 0, \tag{39}$$

$$\tau = \epsilon t, \quad \xi = x - (1 + \epsilon \Pi_0 t), \tag{40}$$

with

$$\Pi_0 = -\frac{(1-\alpha_0)^2 V^2}{2}, \tag{41}$$

$$\Pi_1 = -\left\{2 + \frac{2 + 3\alpha_0\delta[2 + \alpha_0(1-\delta)]}{2\alpha_0(1-\alpha_0-\delta p_{G0})} p_{G0}\right\}, \tag{42}$$

$$\Pi_2 = -\frac{1}{6\alpha_0(1-\alpha_0)}\left\{4\mu + [4\mu + 3\alpha_0(1-\alpha_0)V\varDelta]\left(1 + \frac{\delta p_{G0}}{1-\alpha_0-\delta p_{G0}}\right) + 3\alpha_0\lambda\frac{\delta p_{G0}}{1-\alpha_0-\delta p_{G0}}\right\}, \tag{43}$$

$$\Pi_3 = \frac{\varDelta^2}{6\alpha_0(1-\alpha_0-\delta p_{G0})}. \tag{44}$$

All the constant coefficients in Eq. (39), the nonlinear coefficient Π_1, the dissipation coefficient Π_2, and the dispersion coefficient Π_3, include δ that represents thermal effects. Furthermore, μ and λ are included in only Π_2, that is, both the liquid viscosity and the thermal conductivity affect only the the dissipation of waves.

Figure 1 depicts the dependence of the dissipation coefficient Π_2 on R_0^* for both cases of the present study and the previous study [18]: Π_2 in the present study is smaller than Π_2 in the previous study. Especially, in the case of $R_0^* = 10\,\mu$m, whereas Π_2 in the previous study is about -0.11, Π_2 in the present study is about -0.22, that is, the dissipation effect becomes stronger by the incorporation of the liquid viscosity and thermal conductivity.

Furthermore, we clarify the nonlinear coefficient Π_1 and the dispersion coefficient Π_3 in the present study are larger than those in the previous study. The detailed explanation will be provided in a presentation.

7. SUMMARY

The weakly nonlinear propagation of plane progressive pressure waves in an initially quiescent liquid uniformly containing many spherical microbubbles has been theoretically investigated, especially focusing on the liquid viscosity, the thermal conductivity, and the incorporation of energy conservation equation. From the method of multiple scales, the KdVB equation describing a low frequency long wave has been derived from the basic equations.

As a result, a thermal effect signified by δ influences all the the wave properties, i.e., the nonlinear, dissipation, and dispersion effects. A detailed physics derived by the resultant KdVB equation will be reported in a presentation.

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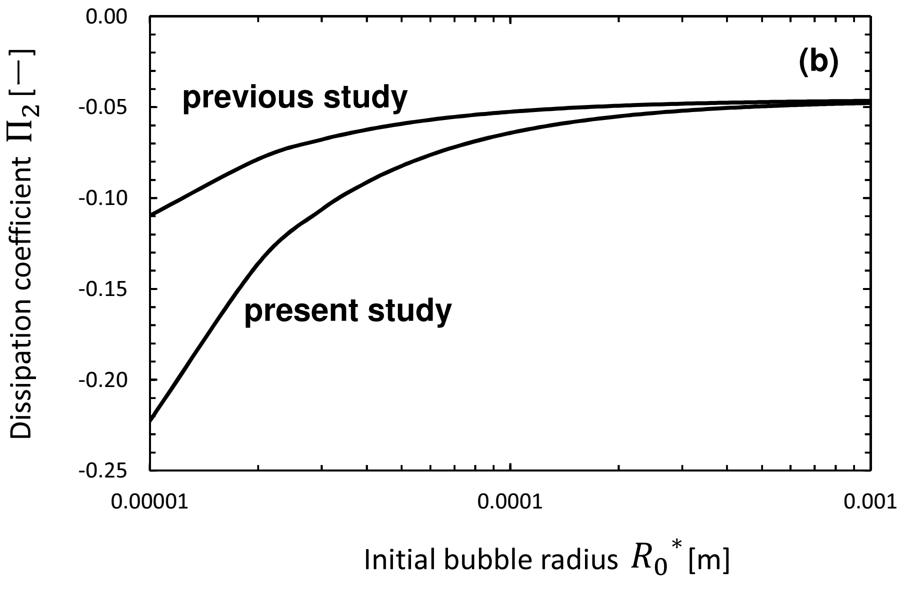

Figure 1: The comparison of dissipation coefficient $\mathit{\Pi}_2$ between the present study and previous study [18], where $\mathit{\Omega} = 1$, $\sqrt{\epsilon} = 0.15$, $\alpha_0 = 0.05$, $p^*_{\mathrm{L0}} = 101325\,\mathrm{Pa}$, $\rho^*_{\mathrm{L0}} = 1000\,\mathrm{kg/m^3}$, $\sigma^* = 0.0728\,\mathrm{N/m}$, $c^*_{\mathrm{L0}} = 1500\,\mathrm{m/s}$, $\mu^* = 1 \times 10^{-3}\,\mathrm{Pa \cdot s}$, $\gamma = 1.4$, $c^*_{\mathrm{P}} = 4.18\,\mathrm{kJ/(kg \cdot K)}$, $\beta^* = 2.06 \times 10^{-4}\,\mathrm{/K}$, and $\lambda^* = 0.598\,\mathrm{W/(m \cdot K)}$.

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