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Two Types of Evolution Equations for Weakly Nonlinear Waves in Bubbly Flows: Effect of Initial Flow Velocity on Pressure Waves

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ABSTRACT

The derivation of two types of effective equations describing an weakly nonlinear propagation of pressure waves in flowing water containing many spherical microbubbles is performed, especially focusing on the effect of an initial flow velocity on wave propagation. For simplicity, the effect of thermal damping is neglected. The basic set of equations is composed of the conservation laws of mass and momentum in a two-fluid model, equation of bubble dynamics, equations of state for each phase, and so on. From the basic set with the aid of the method of multiple scales, we can derive KdVB equation for long-range propagation of a low frequency long wave and the nonlinear Schröedinger equation with an attenuation term for a slowly varying wave packet of moderately high frequency short wave. Finally, we discuss the effect of initial flow velocity on the wave property.

Keywords: Bubbly flow, Bubble dynamics, Pressure wave
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1. INTRODUCTION

Theoretical prediction on nonlinear propagation of pressure waves in liquids containing a number of microbubbles [1–17] is one of most important topics from the viewpoint of multiphase fluid dynamics and nonlinear acoustics. The derivation of nonlinear wave equation describing for a long range propagation of weakly nonlinear waves is effective to predict the wave behavior in a far field. Various nonlinear wave equations such as the Korteweg–de Vries (KdV) equation [1, 2] and the nonlinear Schrödinger (NLS) equation [3] were previously derived. Recently, our group has derived KdV–Burgers (KdVB) equation for a low frequency long wave and the NLS

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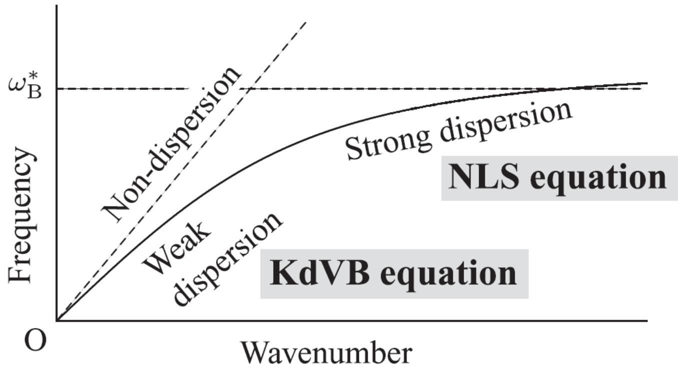

Figure 1: Linear dispersion relation for pressure waves in bubbly liquids [18].

equation for a high frequency short wave via a systematic and unified derivation method (Fig. 1) [18].

However, these results were assumed that the medium is initially quiescent bubbly liquids. Then, the relationship between the wave and a flow has not been clarified. The aim of this paper is to re-derive the KdVB and NLS equations for bubbly flows, that is, the consideration of initial flow velocities of gas- and liquid-phases is the present target.

2. FORMULATION OF THE PROBLEM

This paper deals with an weakly nonlinear propagation of plane progressive waves in a water flows uniformly containing many spherical gas bubbles. The viscosity of the liquid phase is taken into account only at the bubble–liquid interface, although that of the gas phase is omitted. The thermal conductivities of the gas and liquid phases, the Reynolds stress, and the gravitation are dismissed.

Let us use basic equations for bubbly flows proposed by our group [4, 5, 18, 19]. The conservation laws of mass and momentum based on a two-fluid model [4] are written by

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) = 0, \tag{1}$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^*] + \frac{\partial}{\partial x^*}[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*] = 0, \tag{2}$$

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^{*2}) + \alpha\frac{\partial p_{\mathrm{G}}^*}{\partial x^*} = F^*, \tag{3}$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*] + \frac{\partial}{\partial x^*}[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^{*2}] + (1-\alpha)\frac{\partial p_{\mathrm{L}}^*}{\partial x^*} + P^*\frac{\partial\alpha}{\partial x^*} = -F^*, \tag{4}$$

where t^* is the time, x^* space coordinate, α void fraction, ρ^* density, u^* fluid velocity, p^* pressure, and P^* liquid pressure averaged on the bubble–liquid interface [4]; the superscript $*$ represents dimensional quantities; the subscripts G and L represent volume-averaged variables in the gas and liquid phases, respectively.

For the interfacial momentum transport term F^*, the following model of virtual mass force [5, 20] is introduced by

$$F^* = -\beta_1\alpha\rho_{\mathrm{L}}^*\left(\frac{\mathrm{D_G}u_{\mathrm{G}}^*}{\mathrm{D}t^*} - \frac{\mathrm{D_L}u_{\mathrm{L}}^*}{\mathrm{D}t^*}\right) - \beta_2\rho_{\mathrm{L}}^*(u_{\mathrm{G}}^* - u_{\mathrm{L}}^*)\frac{\mathrm{D_G}\alpha}{\mathrm{D}t^*} - \beta_3\alpha(u_{\mathrm{G}}^* - u_{\mathrm{L}}^*)\frac{\mathrm{D_G}\rho_{\mathrm{L}}^*}{\mathrm{D}t^*}, \tag{5}$$

where the values of coefficients β_1, β_2, and β_3 are set as 1/2 for the spherical bubble, and $\mathrm{D_G}/\mathrm{D}t^*$ and $\mathrm{D_L}/\mathrm{D}t^*$ are the Lagrange derivatives.

Keller's equation [21] for spherical symmetric oscillations of a representative bubble in a compressible liquid is given by

$$\left(1 - \frac{1}{c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)R^*\frac{\mathrm{D_G^2}R^*}{\mathrm{D}t^{*2}} + \frac{3}{2}\left(1 - \frac{1}{3c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)\left(\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)^2$$
$$= \left(1 + \frac{1}{c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}\right)\frac{P^*}{\rho_{\mathrm{L0}}^*} + \frac{R^*}{\rho_{\mathrm{L0}}^* c_{\mathrm{L0}}^*}\frac{\mathrm{D_G}}{\mathrm{D}t^*}(p_{\mathrm{L}}^* + P^*), \tag{6}$$

where R^* is a representative bubble radius and c_{L0}^* is the speed of sound in a pure liquid; the subscript 0 denotes the physical quantities in the initial undisturbed state and they are all the constants. Noting that the bubble oscillation is spherically symmetric; the bubbles do not coalesce, break up, disappear, and appear; the effect of direct bubble–bubble interaction is ignored.

The set of Eqs. (1)–(6) is closed by the following supplementary equations: (i) the polytropic equation of state for gas phase,

$$\frac{p_{\mathrm{G}}^*}{p_{\mathrm{G0}}^*} = \left(\frac{\rho_{\mathrm{G}}^*}{\rho_{\mathrm{G0}}^*}\right)^{\gamma}, \tag{7}$$

where γ is the polytropic exponent; (ii) the Tait equation of state for liquid phase,

$$p_{\mathrm{L}}^* = p_{\mathrm{L0}}^* + \frac{\rho_{\mathrm{L0}}^* {c_{\mathrm{L0}}^*}^2}{n}\left[\left(\frac{\rho_{\mathrm{L}}^*}{\rho_{\mathrm{L0}}^*}\right)^n - 1\right], \tag{8}$$

where n is the material constant (e.g., $n = 7.15$ for water); (iii) the conservation equation of mass inside the bubble,

$$\frac{\rho_{\mathrm{G}}^*}{\rho_{\mathrm{G0}}^*} = \left(\frac{R_0^*}{R^*}\right)^3, \tag{9}$$

noting that the gas inside the bubble is composed of only a non-condensable gas such as an air, and hence the phase change across the bubble–liquid interface does not occur; (iv) the balance equation of normal stresses across the bubble–liquid interface,

$$p_{\mathrm{G}}^* - (p_{\mathrm{L}}^* + P^*) = \frac{2\sigma^*}{R^*} + \frac{4\mu^*}{R^*}\frac{\mathrm{D_G}R^*}{\mathrm{D}t^*}, \tag{10}$$

where σ^* is the surface tension and μ^* is the liquid viscosity.

3. MULTIPLE SCALES ANALYSIS

The independent variables t^* and x^* are nondimensionalized as $t = t^*/T^*$ and $x = x^*/L^*$, respectively, where T^* is a typical period and L^* is a typical wavelength. Then the near field and far field are defined by the extended independent variables as multiple scales [22]:

$$t_0 = \epsilon^0 t, \quad x_0 = \epsilon^0 x, \quad \text{(near field)}, \tag{11}$$
$$t_1 = \epsilon^1 t, \quad x_1 = \epsilon^1 x, \quad \text{(far field)}, \tag{12}$$

where ϵ is a nondimensional wave amplitude which is sufficiently small compared with unity (i.e., $0 < \epsilon \ll 1$). The differential operators are then expanded as

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \epsilon \frac{\partial}{\partial t_1}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial x_0} + \epsilon \frac{\partial}{\partial x_1}. \tag{13}$$

Fluid velocities $u_{\rm G}$ and $u_{\rm L}$ are nondimensionalized and expanded in power series of ϵ:

$$\frac{u_{\rm G}^*}{U^*} = u_{\rm G0} + \epsilon u_{\rm G1} + \epsilon^2 u_{\rm G2} + \cdots, \tag{14}$$

$$\frac{u_{\rm L}^*}{U^*} = u_{\rm L0} + \epsilon u_{\rm L1} + \epsilon^2 u_{\rm L2} + \cdots, \tag{15}$$

where U^* is a typical phase velocity, and $u_{\rm G0}$ and $u_{\rm L0}$ are the initial constant gas and liquid velocities, respectively, which have been neglected in the previous study [18]. The limit of $u_{\rm G0} \to 0$ and $u_{\rm L0} \to 0$ corresponds to the previous result [18]. The KdVB and NLS equations derived here are thereby the same as those derived in the previous study [18] for the case that both $u_{\rm G0}$ and $u_{\rm L0}$ are zero.

The expansions of other dependent variables are given by

$$\frac{\alpha}{\alpha_0} = 1 + \epsilon\alpha_1 + \epsilon^2\alpha_2 + \cdots, \tag{16}$$

$$\frac{R^*}{R_0^*} = 1 + \epsilon R_1 + \epsilon^2 R_2 + \cdots, \tag{17}$$

$$\frac{\rho_{\rm L}^*}{\rho_{\rm L0}^*} = \begin{cases} 1 + \epsilon^2\rho_{\rm L1} + \epsilon^3\rho_{\rm L2} + \cdots, & \text{(for KdVB)}, \\ 1 + \epsilon^5\rho_{\rm L1} + \epsilon^6\rho_{\rm L2} + \cdots, & \text{(for NLS)}, \end{cases} \tag{18}$$

$$\frac{p_{\rm L}^*}{\rho_{\rm L0}^* U^{*2}} = p_{\rm L0} + \epsilon p_{\rm L1} + \epsilon^2 p_{\rm L2} + \cdots, \tag{19}$$

where the expansion coefficients of $p_{\rm L}^*$ are defined by $p_{{\rm L}i} = \rho_{{\rm L}i}/V^2$ ($i = 1, 2$).

The nondimensional pressure for the gas and liquid phases in the unperturbed state $p_{\rm G0}$ and $p_{\rm L0}$, the ratio of initial densities of the gas and liquid phases, and the scaling relation of the liquid viscosity are the same as those in the previous study [18].

4. DERIVATION OF KDVB EQUATION

Firstly, we shall derive the KdVB equation for the low frequency long wave. The set of scaling relations appropriate to this case [18] is given by

$$\frac{U^*}{c_{\rm L0}^*} \equiv O(\sqrt{\epsilon}) \equiv V\sqrt{\epsilon}, \tag{20}$$

$$\frac{R_0^*}{L^*} \equiv O(\sqrt{\epsilon}) \equiv \mathit{\Delta}\sqrt{\epsilon}, \tag{21}$$

$$\frac{\omega^*}{\omega_{\rm B}^*} \equiv \frac{1}{T^*\omega_{\rm B}^*} \equiv O(\sqrt{\epsilon}) \equiv \mathit{\Omega}\sqrt{\epsilon}, \tag{22}$$

where V, $\mathit{\Delta}$, and $\mathit{\Omega}$ are the constants of $O(1)$; $\omega^* \equiv 1/T^*$ is a typical angular frequency of incident waves; $\omega_{\rm B}^*$ is the natural angular frequency of linear spherical symmetric oscillations of a single bubble given by

$$\omega_{\rm B}^* \equiv \sqrt{\frac{3\gamma(p_{\rm L0}^* + 2\sigma^*/R_0^*) - 2\sigma^*/R_0^*}{\rho_{\rm L0}^* R_0^{*2}}}. \tag{23}$$

Equations (20)–(22) mean that a typical phase velocity U^* is small compared with the speed of sound in a pure water c_{L0}^*, a typical wavelength L^* is long compared with the representative bubble radius R_0^*, and a typical frequency of incident waves ω^* is low compared with the natural frequency of the bubble ω_B^*, respectively.

Substituting Eqs. (13)–(22) into Eqs. (1)–(10), and then equating the coefficient of like powers of ϵ in the resultant equations, we have the following set of linear equations as the first-order equations: (i) the mass conservation law in the gas phase,

$$\frac{D_G\alpha_1}{Dt_0} - 3\frac{D_G R_1}{Dt_0} + \frac{\partial u_{G1}}{\partial x_0} = 0, \tag{24}$$

(ii) the mass conservation law in the liquid phase,

$$\alpha_0\frac{D_L\alpha_1}{Dt_0} - (1-\alpha_0)\frac{\partial u_{L1}}{\partial x_0} = 0, \tag{25}$$

(iii) the momentum conservation law in the gas phase,

$$\beta_1\left(\frac{D_G u_{G1}}{Dt_0} - \frac{D_L u_{L1}}{Dt_0}\right) + \beta_2(u_{G0} - u_{L0})\frac{D_G\alpha_1}{Dt_0} - 3\gamma p_{G0}\frac{\partial R_1}{\partial x_0} = 0, \tag{26}$$

(iv) the momentum conservation law in the liquid phase,

$$\begin{aligned}(1-\alpha_0)\frac{D_L u_{L1}}{Dt_0} - \alpha_0\beta_1\left(\frac{D_G u_{G1}}{Dt_0} - \frac{D_L u_{L1}}{Dt_0}\right) - \alpha_0\beta_2(u_{G0} - u_{L0})\frac{D_G\alpha_1}{Dt_0}\\ - \alpha_0 u_{L0}\frac{D_L\alpha_1}{Dt_0} + u_{L0}(1-\alpha_0)\frac{\partial u_{L1}}{\partial x_0} + (1-\alpha_0)\frac{\partial p_{L1}}{\partial x_0} = 0,\end{aligned} \tag{27}$$

(v) the Keller equation,

$$R_1 + \frac{\Omega^2}{\Delta^2}p_{L1} = 0. \tag{28}$$

Here, all the partial derivatives with respect to t in the previous study [18] varied the Lagrange derivatives D_G/Dt_0 and D_L/Dt_0 defined by

$$\frac{D_G}{Dt_0} \equiv \frac{\partial}{\partial t_0} + u_{G0}\frac{\partial}{\partial x_0}, \quad \frac{D_L}{Dt_0} \equiv \frac{\partial}{\partial t_0} + u_{L0}\frac{\partial}{\partial x_0}. \tag{29}$$

The initial velocities in Eq. (29), u_{G0} and u_{L0}, are constants, that is, D_G/Dt_0 and D_L/Dt_0 are the linear operators. Although Eqs. (24)–(27) contain the initial velocities, Eq. (28) does not contain the initial velocities. Thus, the bubble oscillations are not affected by the initial velocities in the near field.

From now on, we assume that the velocities are uniform at the initial state (i.e., $u_{G0} = u_{L0} \equiv u_0$). Eliminating α_1, u_{G1}, u_{L1}, and p_{L1} from Eqs. (24)–(28), we have the linear wave equation of R_1,

$$\frac{D^2R_1}{Dt_0^2} - v_p^2\frac{\partial^2 R_1}{\partial x_0^2} = 0, \tag{30}$$

where the phase velocity v_p is given by

$$v_p = \sqrt{\frac{3\alpha_0(1-\alpha_0+\beta_1)\gamma p_{G0} + \beta_1(1-\alpha_0)\Delta^2/\Omega^2}{3\beta_1\alpha_0(1-\alpha_0)}}. \tag{31}$$

Now, we restrict ourselves to the right-running wave, and a phase function φ_0 is introduced as

$$\varphi_0(x_0, t_0) \equiv x_0 - (u_0 + v_\mathrm{p})t_0. \tag{32}$$

Rewriting Eqs. (24)–(28) by φ_0, we express α_1, $u_{\mathrm{G}1}$, $u_{\mathrm{L}1}$, and $p_{\mathrm{L}1}$ in terms of the represented bubble radius R_1.

Let us proceed the derivation of second order of approximation. By the use of the same way as leading order of approximation, we obtain the following set of nonlinear equations:

$$\frac{\mathrm{D_G}\alpha_2}{\mathrm{D}t_0} - 3\frac{\mathrm{D_G}R_2}{\mathrm{D}t_0} + \frac{\partial u_{\mathrm{G}2}}{\partial x_0} = K_1, \tag{33}$$

$$\alpha_0\frac{\mathrm{D_L}\alpha_2}{\mathrm{D}t_0} - (1-\alpha_0)\frac{\partial u_{\mathrm{L}2}}{\partial x_0} = K_2, \tag{34}$$

$$\beta_1\left(\frac{\mathrm{D_G}u_{\mathrm{G}2}}{\mathrm{D}t_0} - \frac{\mathrm{D_L}u_{\mathrm{L}2}}{\mathrm{D}t_0}\right) + \beta_2(u_{\mathrm{G}0} - u_{\mathrm{L}0})\frac{\mathrm{D_G}\alpha_2}{\mathrm{D}t_0} - 3\gamma p_{\mathrm{G}0}\frac{\partial R_2}{\partial x_0} = K_3, \tag{35}$$

$$\begin{aligned}(1-\alpha_0)\frac{\mathrm{D_L}u_{\mathrm{L}2}}{\mathrm{D}t_0} - \alpha_0\beta_1\left(\frac{\mathrm{D_G}u_{\mathrm{G}2}}{\mathrm{D}t_0} - \frac{\mathrm{D_L}u_{\mathrm{L}2}}{\mathrm{D}t_0}\right) - \alpha_0\beta_2(u_{\mathrm{G}0} - u_{\mathrm{L}0})\frac{\mathrm{D_G}\alpha_2}{\mathrm{D}t_0} - \alpha_0 u_{\mathrm{L}0}\frac{\mathrm{D_L}\alpha_2}{\mathrm{D}t_0}\\ + u_{\mathrm{L}0}(1-\alpha_0)\frac{\partial u_{\mathrm{L}2}}{\partial x_0} + (1-\alpha_0)\frac{\partial p_{\mathrm{L}2}}{\partial x_0} = K_4,\end{aligned} \tag{36}$$

$$R_2 + \frac{\Omega^2}{\varDelta^2}p_{\mathrm{L}2} = K_5, \tag{37}$$

where the inhomogeneous terms K_i's are composed of the partial derivatives of the first-order perturbations:

$$K_1 = 3\frac{\mathrm{D_G}R_1}{\mathrm{D}t_1} - \frac{\mathrm{D_G}\alpha_1}{\mathrm{D}t_1} - \frac{\partial u_{\mathrm{G}1}}{\partial x_1} + 3\frac{\mathrm{D_G}\alpha_1 R_1}{\mathrm{D}t_0} - 12R_1\frac{\mathrm{D_G}R_1}{\mathrm{D}t_0} + 3\frac{\partial R_1 u_{\mathrm{G}1}}{\partial x_0} - \frac{\partial \alpha_1 u_{\mathrm{G}1}}{\partial x_0}, \tag{38}$$

where K_i's $(2 \le i \le 5)$ are omitted due to space limitations.

In the same way as that used in the derivation of Eq. (30), we derive the following inhomogeneous equation for R_2,

$$\frac{\mathrm{D}^2R_2}{\mathrm{D}t_0^2} - v_\mathrm{p}^2\frac{\partial^2 R_2}{\partial x_0^2} = K, \tag{39}$$

where the inhomogeneous term K is given by

$$K = -\frac{1}{3}\frac{\mathrm{D}K_1}{\mathrm{D}t_0} + \frac{1}{3\alpha_0}\frac{\mathrm{D}K_2}{\mathrm{D}t_0} + \frac{u_0}{3\alpha_0(1-\alpha_0)}\frac{\partial K_2}{\partial x_0} + \frac{1-\alpha_0+\beta_1}{3(1-\alpha_0)\beta_1}\frac{\partial K_3}{\partial x_0} + \frac{1}{3\alpha_0(1-\alpha_0)}\frac{\partial K_4}{\partial x_0} - \frac{\varDelta^2}{3\alpha_0\Omega^2}\frac{\partial^2 K_5}{\partial x_0^2}. \tag{40}$$

Rewriting Eq. (40) by using Eq. (32), we have

$$K = 2v_\mathrm{p}\frac{\partial}{\partial\varphi_0}\left[\frac{\partial R_1}{\partial t_1} + (v_\mathrm{p} + u_0)\frac{\partial R_1}{\partial x_1} + \Pi_0\frac{\partial R_1}{\partial\varphi_0} + \Pi_1 R_1\frac{\partial R_1}{\partial\varphi_0} + \Pi_2\frac{\partial^2 R_1}{\partial\varphi_0^2} + \Pi_3\frac{\partial^3 R_1}{\partial\varphi_0^3}\right], \tag{41}$$

where the coefficients Π_i $(0 \le i \le 3)$ are given by

$$\Pi_0 = -\frac{v_\mathrm{p}(1-\alpha_0)\varDelta^2 V^2}{6\alpha_0\Omega^2}, \quad \Pi_2 = -\frac{1}{6\alpha_0}\left(4\mu + \frac{V\varDelta^3}{\Omega^2}\right), \quad \Pi_3 = \frac{\varDelta^2}{6\alpha_0}, \tag{42}$$

the explicit form of Π_1 is very complex and not shown for the economy of space.

The non-secular condition for Eq. (39) gives $K = 0$. By using Eqs. (13) and (30), the independent variables φ_0, t_1, and x_1 in Eq. (41) are restored into t and x, and we have

$$\frac{\partial R_1}{\partial t} + (v_\mathrm{p} + u_0)\frac{\partial R_1}{\partial x} + \epsilon\left[\Pi_0\frac{\partial R_1}{\partial x} + \Pi_1 R_1\frac{\partial R_1}{\partial x} + \Pi_2\frac{\partial^2 R_1}{\partial x^2} + \Pi_3\frac{\partial^3 R_1}{\partial x^3}\right] = 0. \tag{43}$$

Finally, via the variables transform

$$\tau \equiv \epsilon t, \quad \xi \equiv x - (v_\mathrm{p} + u_0 + \epsilon\Pi_0)t, \tag{44}$$

the KdVB equation can be derived:

$$\frac{\partial R_1}{\partial \tau} + \Pi_1 R_1\frac{\partial R_1}{\partial \xi} + \Pi_2\frac{\partial^2 R_1}{\partial \xi^2} + \Pi_3\frac{\partial^3 R_1}{\partial \xi^3} = 0, \tag{45}$$

where Π_1 is the nonlinear coefficient, Π_2 is the dissipation coefficient, and Π_3 is the dispersion coefficient.

The initial flow velocity u_0 is not contained in Π_1, Π_2, and Π_3. On the other hand, the transformed space coordinate ξ contains u_0. We then conclude that the initial flow velocity does not affect the nonlinear, dissipation, and dispersion properties. It is implied that the wave is independent of a flow under the uniform flow assumption in Eqs. (14) and (15).

5. NLS EQUATION

the derivation of the NLS equation for the envelope wave of moderately high frequency short carrier wave is briefly explained. The set of scaling relations [18] is given by

$$\frac{U^*}{c^*_{\mathrm{L0}}} \equiv O(\epsilon^2) \equiv V\epsilon^2, \tag{46}$$

$$\frac{R^*_0}{L^*} \equiv O(1) \equiv \mathit{\Delta}, \tag{47}$$

$$\frac{\omega^*}{\omega^*_\mathrm{B}} \equiv O(1) \equiv \mathit{\Omega}. \tag{48}$$

The resultant NLS equation is as follows:

$$\mathrm{i}\frac{\partial A}{\partial \tau} + \frac{1}{2}\frac{\mathrm{d}v_\mathrm{g}}{\mathrm{d}k}\frac{\partial^2 A}{\partial \xi^2} + C_1|A|^2 A + \mathrm{i}C_2 A = 0, \tag{49}$$

with

$$\tau = \epsilon^2 t, \quad \xi = \epsilon[x_0 - (v_\mathrm{g} + u_0)t_0], \tag{50}$$

where τ and ξ are the transformed time and space coordinate, A is the complex amplitude of carrier wave, v_g is the group velocity, C_1 and C_2 are the real constants. As in the case of KdVB equation, it is implied that the envelope wave A is independent of a fluid flow u_0.

6. SUMMARY

Weakly nonlinear propagation of pressure waves in bubbly flows with initial velocities was theoretically investigated. Both the KdVB equation for a long wave and the NLS equation for a short carrier wave were derived. As a result, the initial uniform flow velocity u_0 affects the transformed space coordinate, and the nonlinear, dissipation, and dispersion coefficients in the KdVB and NLS equations are not affected by u_0.

In a presentation, a detailed physics predicted the resultant KdVB and NLS equations, the case of initially non-uniform case (i.e., $u_{G0} \neq u_{L0}$), and another perturbation expansion in Eqs. (14) and (15) such as the case of an arbitrary function with respect to space [e.g., $u_{G0} = u_{G0}(x_1)$], will be described.

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