MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

Construction of Large and Accurate Energy-based Vibroacoustic Models using Vibrational Intensity Approach Combined to Virtual SEA Analysis

Borello Gérard[1]
Borello Romain[2]

InterAC
10 impasse Borde Basse, ZA La Violette, 31240 L'Union, France

ABSTRACT

Broadband frequency energy-based industrial predictive methods are mostly related to Statistical Energy Analysis (SEA). Despite well-proven efficiency in predicting acoustic environment in trimmed vehicles, SEA applied to the prediction of vibratory transfers in large systems (buildings, cruise ships and aircrafts) shows inherent drawbacks related to too coarse assumptions in deriving the relevant SEA parameters required by the modelling. Using diffuse intensity in place of energy as unknowns, is shown to relax the most constraining SEA assumptions and allows energy-based models to be better conditioned for large size (i.e. large number of subsystems). This methodology combined with the Virtual SEA method for smooth description of local heterogeneity, expands the range of applications of current energy-based methods to large and complex industrial systems as well as the frequency bandwidth of model validity while simplifying model construction.

Keywords: Noise, Environment, SEA, Intensity, VSEA
I-INCE Classification of Subject Number: 76

1. INTRODUCTION

Statistical Energy Analysis (SEA) is providing a theoretical framework for random prediction of vibroacoustic environment applicable to a wide class of industrial systems from basic domestic appliance up to sophisticated aircraft and spacecraft. From this, SEA has been integrated as part of the design process in many domains of the industry. This need has originated several software implementing the SEA methodology, essentially based on the conservation of energy flow between the various subsystems involved in the analysis. Lyon and Maidanik original work [1] is based on a modal formulation of energy flow exchange, but SEA has rather evolved to a wave-transmission approach in diffuse-field conditions, due mainly to the difficulty in relating the energy flow coefficients (the Coupling Loss Factors or CLF) and the modal mechanical coupling coefficients derived from mass and stiffness matrices of classical dynamic equation equilibrium [14].

[1] gerard.borello@interac.fr
[2] romain.borello@interac.fr

SEA theory is based on a weak-coupling assumption between subsystems, which is most of the time satisfied in the high frequency regime (i.e. for short wavelengths propagating in any subsystem). With this assumption, a subsystem is only depending on its local modes and may be easily computed using standard analytical solutions for a given choice of typology in describing the subsystem as a beam, a plate or a more complex curved-shell.

Coupling subsystems together for deriving the CLF cannot be easily done within the classical modal theory or at the price of introducing new set of assumptions. For any dynamic problem based on substructuration, the domain of analysis has to be split into parts, each part being described by modal series of pre-computed modes.

The behavior of the full system is obtained by matrix assembly of the various subsystem sub-matrices. The assembly method is heavily depending on the choice of boundary conditions allocated to individual subsystems to extract their modal behavior: free, clamped, mix of the latter or adding some additional degrees of freedom as in the Craig-Bampton assembly method [2]. Latter choices condition the convergence of the final modal-series matrix to a solution. The modal coupling loss factors between pair of modal oscillators taken between two different subsystems is entirely dependent on assumed boundary conditions and only the assembled matrix has a physical meaning. Many references can be found that have discussed about possible ways of retrieving CLF for modal coupling but none has evolved to a general applicable light calculation method [9][11][1211].

2. CLASSICAL ANALYTICAL SEA DERIVATION OF CLF BETWEEN CONTINUOUS AND CONTIGUOUS SUBSYSTEMS

As proposed by Lyon [2], the CLF are simply obtained between two continuous domains using both weak-coupling and diffuse-field assumptions.

The field diffusion (in high frequency range) allows replacing the modal representation of the vibratory field by a set of uncorrelated plane waves within the emitter subsystem. The weak-coupling assumption guarantees the energy that flows into the receiver through a junction (point, line, area...) is not feed-backed into the emitter as correlated energy.

On one hand, the power flower $P_{E->R}$ between the emitter and the receiver subsystems, due a diffuse total energy state E_E, within the emitter in a frequency band centered around the radian frequency ω, is simply expressed as function of energy:

$$P_{E->R} = \eta_{ER} \omega E_E$$

On the other hand, the power flow may be obtained from the transmission coefficient by introducing the diffuse field intensity in the emitter, I_d that crosses the junction of size L_j:

$$P_{E->R} = \langle \tau \rangle_\Omega I_d L_j$$

From two previous equations and given the ratio of diffuse intensity over total energy in the emitter medium, it comes:

$$\eta_{ER} = \frac{\langle \tau \rangle_\Omega}{\omega} \frac{I_d L_j}{E_E} \tag{1}$$

with $\frac{I_d}{E_E}=\frac{c_g}{2L}$ for beam-to-beam coupling; $\frac{I_d}{E_E}=\frac{c_g}{\pi A}$ for plate-to-plate coupling and $\frac{I_d}{E_E}=\frac{c_g}{4V}$ for cavity-to-cavity coupling. *L, A and V* are respectively emitter length, area and volume depending on subsystem typology.

Group speed is only different from phase-wave speed for dispersive waves (reduced to flexural structural waves in standard SEA applications):

$$c_g = 2c_f = 2\sqrt{\frac{\mathsf{E}}{\rho}}$$

If the two coupled subsystems are both driven by a random force, both forces being uncorrelated, the net power flow at their interface under weak-coupling assumption is given by:

$$P_{12} = \eta_{12}\omega E_1 - \eta_{21}\omega E_2 \qquad (2)$$

Using the wave transmission coefficient provided by (1), independent of damping loss factor, leads obviously to inconsistency when the transmission loss is large (i.e. near to 1). For $\langle\tau\rangle \approx 1$, the energy sent by 1 to 2 is sent back to 1 in an amount that cannot be neglected and not fulfilling the weak-coupling criterion.

Previous considerations explain why regular SEA method is more suitable for solving fluid-structure interactions than structure borne problems. Transmission loss coefficient between two cavities separated by a light panel is generally of around 0.01 or lower; weak-coupling is implicitly satisfied and equation (2) applies. In mechanical coupling the transmission loss is most often spread between 1 and 0.1, obviously not a weak-coupling case and some bias is expected from actual behavior when chaining similar subsystems and predicting the transfer from (2).

3. VIRTUAL SEA MODELING FOR LOW AND MID-FREQUENCIES

By applying Analytical SEA (ASEA) to the modelling of mechanical transfers in car body-in-white (BIW) [8][13], it was observed that ASEA CLF were deviating from measurements in the mid-frequency range. SEA parameters were extracted from test data and inverse SEA test method, also called the Power Injection Method (PIM) and compared to predicted ones.

Outputs from analytical models were also showing high sensitivity to the way BIW or chassis were partitioned into subsystems. The Virtual SEA (VSEA) method has been then introduced from 2000th's to determine SEA parameters from Finite Element models (FEM) and was proved to be an efficient way of building truly predictive mid-frequency energy models of car bodies [13].

In VSEA, SEA models are created from FEM by processing Frequency Response Functions (FRF) computed by modal synthesis using the FEM global modes using an adaptation of PIM method to specificity of FEM outputs. The major benefit of VSEA is the auto-partition technique that identifies the subsystem domain extension as a function of frequency in a given frequency band of analysis, to guarantee two subsystems are effectively weakly coupled for equation (2) to be applicable.

VSEA provides a compressed model of the FEM statistical dynamics up to the frequency limit of modal extraction (1/6 of the wavelength) under the SEA format (i.e. a frequency and size dependent loss factor matrix, coupling energy and power).

VSEA is also a handy tool to compare ASEA models and their related FEM representations.

The following example in Figure 1 shows a “box” FEM model and Figure 2 the resulting SEA model post-processed by VSEA beside the related regular ASEA model.

The FEM FRF from which is identified the VSEA model were synthesized with a global modal DLF of 0.01. The “direct” junction transmission coefficients range between 0.2 and 0.7 whether adjacent plates are orthogonal or coplanar (analytical calculation).

When applying a force on the first transverse panel, the evolution of the VSEA lateral panel velocities is given in Figure 3 and the related velocities computed by the analytical SEA model are given in Figure 4. We note a progressive divergence of levels with distance to source given by the analytical model compared to FEM/VSEA result. This divergence is also observed on transverse panel levels.

The depredation of the analytical modelling comes from the inadequacy of equation (2) in expressing the power flow exchanged between panels due to the strength of coupling in the concerned frequency region of interest.

This is why VSEA has been widely used in place of ASEA for simulating mid-frequency behavior of various industrial systems by embedding both structure and acoustic-borne sound propagation within the same model.

Nevertheless, there is a need for some kind of analytical modelling for solving large systems such as aircraft or ship. The bandwidth of their vibrational responses is limited due to the decrease of FEM upper frequency limit with model size. ASEA models are biased for strong coupling and serial path propagation over low and mid-frequencies as transmission coefficients of structural elements stay nearly constant with frequency within this frequency range.

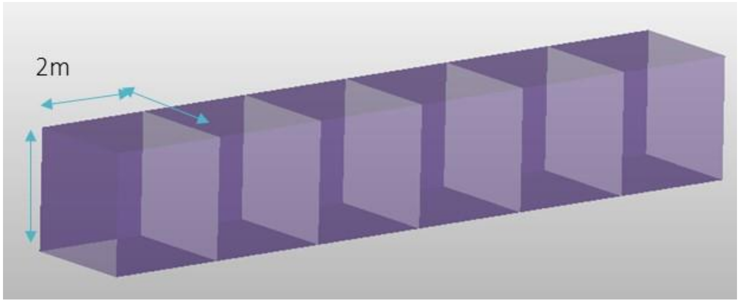


Figure 1: FEM box model: Elementary panel 2 m x 2 m in steel 1 cm thick. Mesh size 30 mm

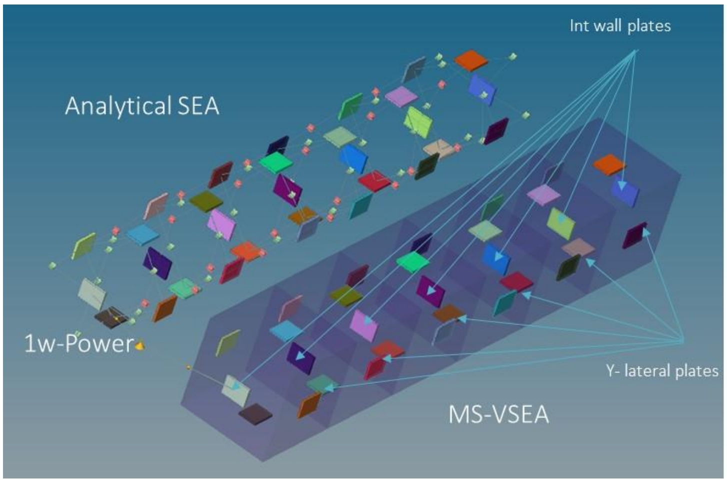


Figure 2: MS-VSEA and Analytical SEA box models

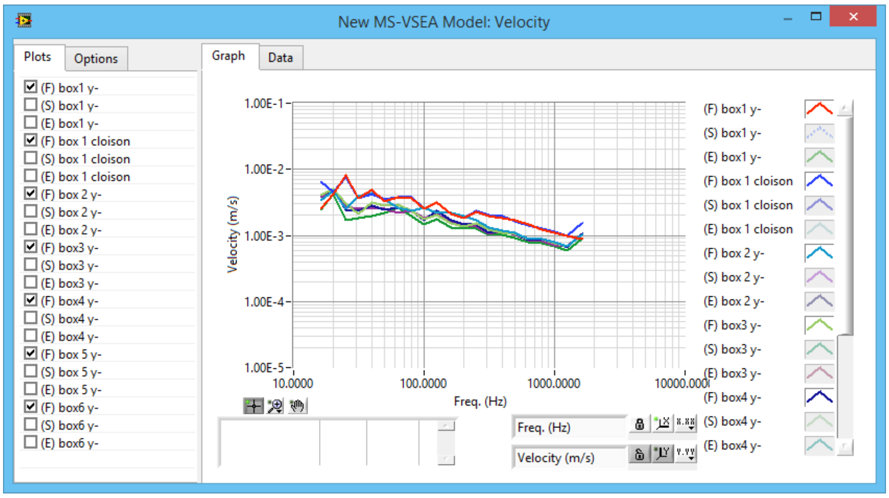


Figure 3: Evolution of lateral-plate velocity with distance from source (first transverse plate) in the VSEA model

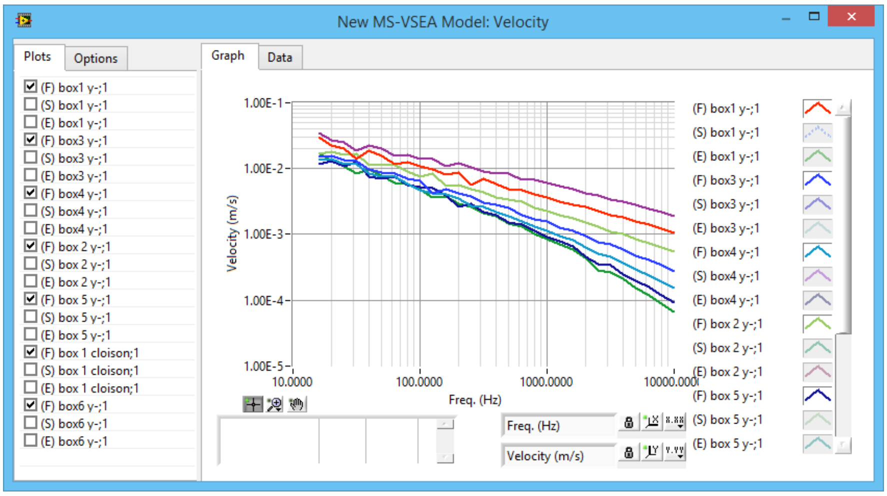


Figure 4: Evolution of lateral-plate velocity with distance from source (first transverse plate) in the analytical SEA model

4. INTENSITY METHOD FOR SOLVING STRONG-COUPLING CASE

Strong coupling is the most common case when modelling a structural industrial system as all parts have similar materials and cross-sections as in aircrafts, ships or cars.

Applying formula (1) to mechanical junctions in an ASEA model, leads to a bias in predicted CLF and error cumulates when chaining these subsystems as shown in previous box example.

From energy flow analysis [3][4][6] and author work [5], an alternate expression of the conservation of energy exists. This expression uses diffuse intensities at subsystem boundary as unknowns in place of the subsystem energy.

The net power flow that crosses a junction connecting two subsystems can be split into two contributions: the power that flows out of the subsystem, with positive sign referring to the outward normal at subsystem boundary and the power that flows in, with negative sign.

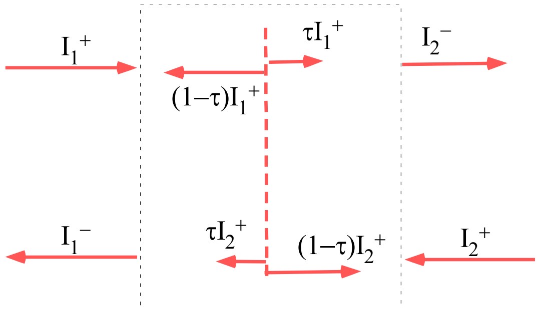
The net power flow expresses then at boundary *j* as: $P_j = \left(I_j^+ - I_j^-\right)L_j$ and I_j^+, I_j^- are called the circulating intensities for differencing them from net intensity, the standard measured quantity. A subsystem may have several boundaries and from [5] it is shown that circular intensities at boundary *j* are related to other intensities at other boundaries *i* by the following expression:

$$I_j^+ = \sum_i \alpha_{ij} I_i^- e^{-\frac{\eta\omega}{c_g} d_{ij}} \tag{3}$$

where d_{ij} is the mean free run distance between two boundaries and α_{ij} the geometrical divergence coefficient depending on subsystem shape.

For a non-dissipative junction connecting two subsystems 1 and 2, the net power flow crossing it depends on circular intensity balance at boundary and next two relationships express the conservation of intensity as function of the diffuse wave-transmission coefficient, τ :

$$\begin{cases} I_{(2)}^- = \tau I_{(1)}^+ + (1-\tau) I_{(2)}^+ \\ I_{(1)}^- = \tau I_{(2)}^+ + (1-\tau) I_{(1)}^+ \end{cases} \tag{4}$$



From equations (3) and (4), circulating intensities at boundaries of a network of subsystems can be linked mathematically in a matrix form to injected power in the system for solving the intensity problem, leading to the Statistical Intensity Analysis (SIA) method.

SIA uses more unknowns than SEA, at least two circulating intensities per boundary (or connection) plus a certain number of free boundaries where either perfect reflection or absorption can be stated. Unlike SEA, boundaries can be made dissipative when needed.

It also solves the problematic of weak coupling as equation (4) does not involve any restriction to the value of τ . Best, assuming $\tau << 1$, equations (4) simplify and lead to the regular SEA expression (2), SEA being proved to be included within the SIA theoretical framework.

Energy density within subsystems can be retrieved from boundary intensities after solve as their expression is found given by:

$$\langle e_d(\mathbf{x})\rangle = \frac{1}{L}\int_L e(\mathbf{x})d\mathbf{x} = F\left(\eta k_g \mathbf{l}\right)\sum_i \left\{\left|I_i^+\right| + \left|I_i^-\right|\right\}$$

F is a continuous function obtained by integrating the shape function mapping the subsystem domain defined by series of boundaries B_i .

The shape function describes the attenuation of circulating intensities across the subsystem. It depends on characteristic subsystem length, $\mathbf{l}$, group wavenumber k_g and damping loss factor η .

SIA shares common assumptions with SEA: each subsystem must be resonant within the frequency band of analysis which means the structural wavelength should be shorter than the subsystem characteristic size.

To compare the virtue of SIA against VSEA and ASEA, a two-plate subsystem case is now introduced: the two plates are made of aluminum, their thickness is 2 mm with dimensions 1 m x 1 m and they are connected by a 1 m-line junction. The diffuse transmission coefficient is equal to 0.28 as analytically computed by SEA+ software [15].

The plate models are built using ASEA and VSEA modeling methods, the latter relying on a FEMAP FEM model of which modes were extracted up to 9000 Hz and modal amplitudes stored at 143 reference nodes. FEM modal amplitude file is submitted to the SEAVirt solver of SEA+ which performs the FRF computation between nodes and identifies SEA parameters from FRF using PIM method. The output of SEAVirt provides automatically a VSEA model of the two-plate system able to predict within 0.5 to 1 dB of uncertainty the plate transfer energies.

The three models are respectively solved for a 3-Watts power applied in subsystem 1 and for three different values of modal DLF (same for both plates), respectively 0.1%, 1% and 10%.

Comparisons are made on ratio $P_{12} / \omega E_{11}$ to emphasize the differences between the three models vs. DLF as differences in energies are small as expected for such a simple model. P_{12} is the net power flow crossing the junction and E_{11} the energy of the emitter subsystem.

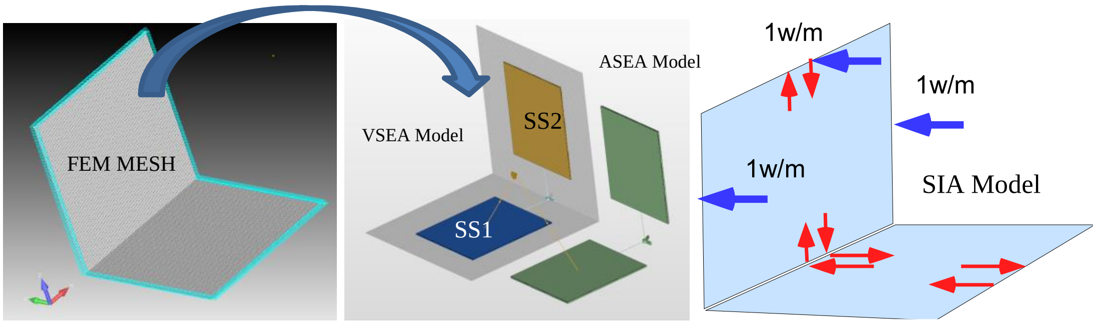


Figure 5: 2-Plate test case modeled by VSEA, ASEA and SIA

Figure 6 and Figure 7 show outputs from the three models for the three DLF values. SIA is closer to VSEA/FEM output over mid-frequency compared to ASEA.

This offset explains why chaining strongly-coupled subsystems leads to cumulative errors. SIA offers a better conditioned method with clear separation between damping and transmission effects as the coupling between two subsystems at common boundary depends only on the transmission coefficient, independent of damping.

This is not the case of the ASEA coupling as the CLF is made independent of damping adding an extra assumption of weak coupling not always true.

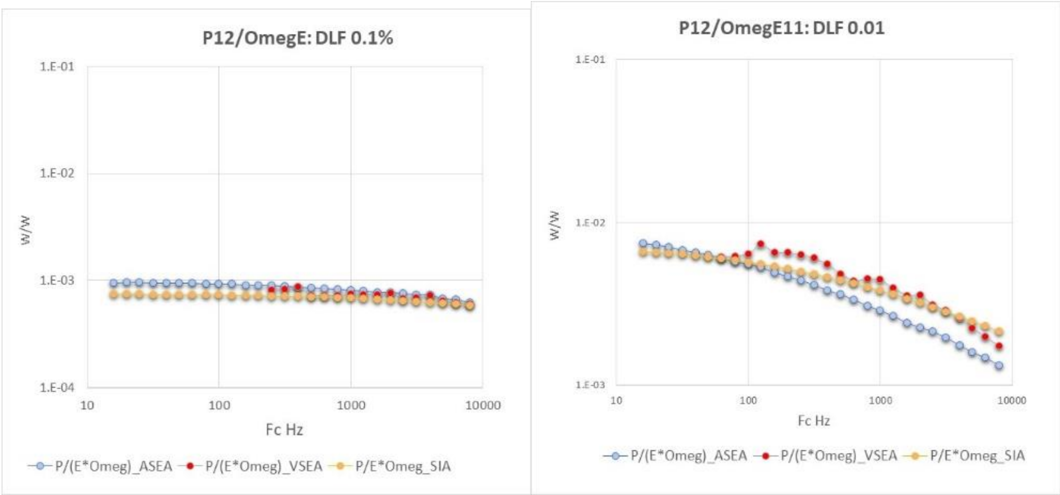


Figure 6: 2-Plate test case Ratio $P/\omega E$ computed by SIA, VSEA and ASEA models for DLF value of 0.1% (left) and 1% (right)

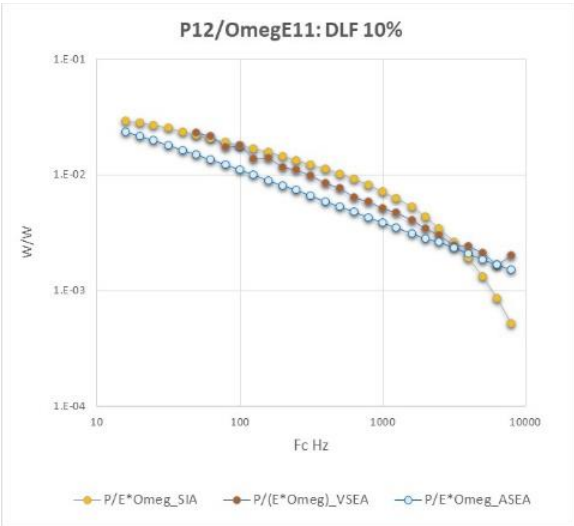


Figure 7: 2-Plate test case Ratio $P/\omega E$ computed by SIA, VSEA and ASEA models for DLF value of 10%

5. CONCLUSIONS

Statistical Intensity Analysis or SIA is suitable to the analysis of structural transmission in case of strong mechanical coupling. SIA intensity equilibrium provides a wider scope than analytical SEA. The classical SEA specific power balance expression (as function of energy difference) is shown to be a specific weak coupling case of the SIA theory.

SIA may then address modeling larger systems than SEA, made of many similar parts chained together like ship hull or full-aircraft fuselage. It nevertheless needs larger number of unknowns for solving then SEA (typically 8 intensities for QUAD SIA elements).

Current work is to hybridize SIA, VSEA and ASEA in a global SEA network model for preserving SEA fast solve capability while improving robustness of structure-borne increasing the scope of analytical calculation.

6. REFERENCES

1. R. H. Lyon and G. Maidanik, *"Power Flow between Linearly Coupled Oscillators",* JASA. Vol.34, (5), pp 623-639 (1962)
2. Roy Craig and Mervyn Bampton *"Coupling of Substructures for Dynamic Analyses"* AIAA Journal, Vol. 6, No.7, pp. 1313-1319 (1968)
3. D.J. Nefske, "*Power Flow finite element analysis of dynamic systems: basic theory and applications"*, SAE (1987)
4. O. Bouthier and R. Bernhard, *"Energy and Structural Intensity Formulations of Beam and Plates",* 3rd Conference on Intensity Techniques, Senlis (1990)
5. G. Borello, *"Modélisation Haute Fréquence du Comportement Dynamique d'un Treillis de poutre",* Rapport Technique RT/CER/003 pour le CERDAN (1992)
6. A. Carcaterra and A. Sestieri, *"Energy Density Equations and Power Flow in Structures",* JSV, 188 (2), pp 269-282 (1995)
7. R. H. Lyon and Dejong, *"Statistical Energy Analysis",* Sd Edition Butterworth-Heinemann (1995)
8. G. Borello, *"Prediction and Control of Structure Borne Noise Transfers in Vehicles Using SEA",* Euro-Noise, Munich, Germany (1998)
9. R.S. Langley and P. Bremner, *"A Hybrid Method for the Vibration Analysis of Complex Structural Acoustic Systems",* JASA, Vol. 105 (3), pp.1657-1671 (1999)
10. G. Borello, L. Gagliardini, L. Houillon and L. Petrinelli, *"Virtual SEA: Mid-Frequency Structure-Borne Noise Modeling Based on Finite Element Analysis",* SAE Noise and Vibration Conference, Traverse City, Michigan, USA (2003)
11. L. Guyader and N. Totaro, "*Structural Partitioning and Power Flow Analysis"*, Inter-Noise, Prague, Czech Republic (2004)
12. X. Zhao and N. Vlahopoulos, *"A Basic Hybrid Finite Element Formulation for Mid-Frequency Analysis of Beam Connected at an Arbitrary Angle",* JSV, Vol. 269 (6) (2004)
13. G. Borello, L. Gagliardini, L. Houillon and L. Petrinelli, *"Virtual SEA-FEA-13Based Modeling of Structure-Borne Noise",* Sound and Vibration Magazine (2005)
14. G. Borello, *"Analyse statistique énergétique SEA & Applications industrielles de la SEA"*, Techniques de l'ingénieur, R 6 215 & R 6 216 (2006) et R 6 215**v**2 (mis à jour/Update 2012)
15. InterAC sarl, *"SEA+ software User and Theoretical Manuals*" (2018)