

An Analytical Model for Broadband Sound Transmission Loss of a Finite Single Leaf Wall using a Metamaterial

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ABSTRACT

Acoustic metamaterials (AM) have emerged as an academic discipline within the last decade. When used for sound insulation, metamaterials can show high transmission loss at low frequencies despite having low mass per unit area. This paper investigates the possibility of using AMs for increasing the sound insulation of finite single leaf walls (SLW), focusing on the coincidence effect problem. Formulas are derived using a variational technique for the forced sound transmission of finite SLW with a coupled array of single degree of freedom resonators. An analytical model is presented for this simple case, and the effects of the band gap in sound transmission and radiation are analysed. Moreover, the influence of each parameter is studied giving way to an optimized way of designing this type of structures using constrained parameter optimization. Different objective functions are compared and discussed. Finally, some conclusions are drawn regarding the effectiveness of the proposed model, possible applications, and future work.

Keywords: Sound-Insulation, Metamaterial, Analytical
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1. INTRODUCTION

The importance of sound insulation has increased in cities with the ever-growing population. Buildings are in closer proximity to each other. Also growing number of vehicles has given rise to noise pollution in cities, which in-turn has necessitated sound insulation in buildings. In offices, it is essential to keep the noise level at the minimum to enhance employee efficiency [1-3]. In schools, audio comfort is one of the primary conditions necessary for an effective learning environment. If there are background noises

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or a high decibel rating inside a classroom, students could struggle to hear adequately, or may find discomfort from straining to hear. It is also easier to become distracted [4].

Acoustic metamaterials have emerged as an academic discipline within the last decade. The definition may be broadly interpreted as systems or materials that display (as a whole) extraordinary properties not found in natural materials with respect to sound and vibration characteristics, such as negative apparent mass and/or bulk modulus. Metamaterials can show high transmission loss (TL) at low frequencies despite having low mass per unit area [8-13]. They owe this behaviour to internal subwavelength periodic structures. One of the most important characteristics of the AM is the so-called band gaps (BG), a frequency region where wave propagation is not possible. This property shows great promise to be a good tool to be used in sound insulation, absorption, and even radiation. Sound insulation of walls in buildings or vehicles is a broadband problem, and for a single homogenous structure, the sound insulation is mainly given by the mass per unit area of the wall, which leaves not much room for improvement [6-7]. There are however a few possible problems that are well suited for the use of acoustic metamaterials, namely the low frequency resonance in double wall constructions, wave propagation and flanking transmission of walls periodically reinforced by beams, and the coincidence effect. Band gaps can be introduced into these structures by mounting an array of resonators to them. This type of construction has been studied and validated in recent years [8-13].

This paper investigates the possibility of using AMs for increasing the sound insulation of single leaf walls, focusing on the coincidence effect problem. The approach utilized in this paper is the same as Brunskog's when investigating the forced sound transmission of single leaf walls using a variational technique [16]. In the present research simple formulas are derived for the forced sound transmission of a finite single leaf wall with a coupled array of single degree of freedom resonators. An analytical model is presented for this simple case, and the effects of the band gap in sound transmission and radiation are analysed. The developed model is restricted to the low frequency range where the wavelengths of the wall is much longer than the periodic distance of the resonators. Moreover, the influence of each parameter is studied, giving way to a possible optimized way of designing these type of structures.

2. THEORY

This section will present the theory utilized in this paper. The variational formulation of the problem used throughout this study is based on Brunskog's work [16] and is extended for the case described in the following section.

2.1 Problem description

Consider a finite thin plate with mass per unit area m_p'' lying in the x - y plane coupled with periodically attached resonators as seen in figure 1a. The plate is located inside a rigid baffle at $z = 0$. For $z < 0$ the acoustic field consists of an incident plane wave p_i , a reflected plane wave p_r , and one scattered field p_s due to the motion of the finite wall. On the positive side of z only the transmitted wave is present (p_t). The resonators have mass per unit area m_r'' and stiffness per unit area s'' . Structural damping of the spring is considered by assigning the inherent losses to the spring element. For harmonic motion this can be represented by a complex stiffness $\underline{s}'' = s''(1 + i\eta_s)$ where η_s is the damping loss factor and s'' is the real part of the complex spring constant. It is of interest to analyse the transmission through this structure and the influence of the resonators.

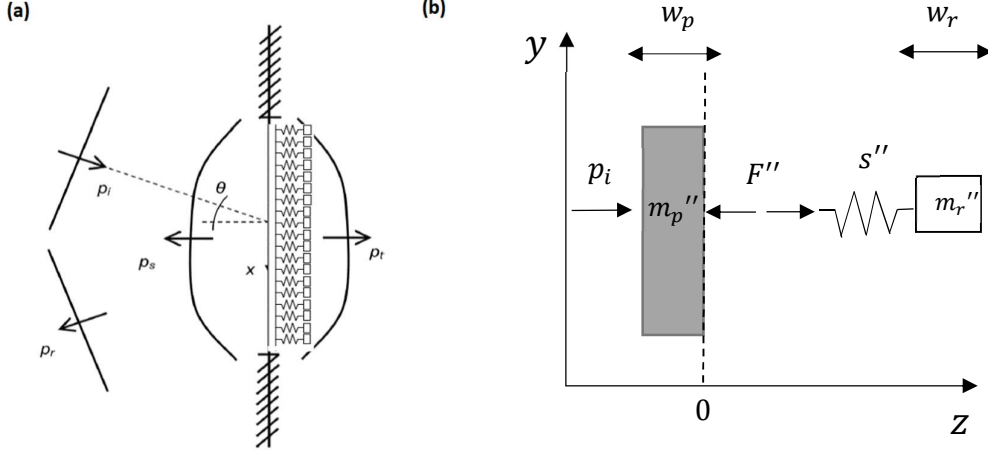


Figure 1: (a) A finite wall of dimensions $a \times b$ coupled with a series of mass-spring resonators located inside a rigid baffle in the x - y plane, at $z=0$. (b) Simplified diagram of a small section

2.2 Model development

Considering the described problem, it is pertinent to start from the forced Helmholtz equation for bending waves in plates. This describes a plate being excited by an external field with pressure distribution $p_i(x, y)$

$$\nabla^4 w_p(x, y) - k_b^4 w_p(x, y) = \frac{p_i(x, y)}{B'} , \quad (1)$$

where k_b is the wavenumber of the free bending wave in the plate, and B' the bending stiffness of the plate. With this notation, $w_p(x, y)$ corresponds to traverse displacement. Note that $\nabla^4 = \nabla^2 \nabla^2$ is the bi-harmonic operator and ∇^2 is the Laplace operator.

As we can see in figure 1b, the resonators will behave as motion excited resonators. The displacement of the plate and the displacement of the mass of the resonators are related by

$$w_r(x, y) = w_p(x, y) \frac{s''(1+i\eta_s)}{m_r''(\omega_0^2 - \omega^2) + i\eta_s s''} , \quad (2)$$

where ω_0 represents the natural resonant frequency of the mass-spring system being

$$\omega_0 = \sqrt{\frac{s''}{m_r''}} . \quad (3)$$

Using Equation 1 and 2 the following expression can be developed

$$\nabla^4 w_p(x, y) - \left(k_b^4 + \frac{m_r'' \omega^2 s'' (1+i\eta_s) / B'}{m_r'' (\omega_0^2 - \omega^2) + i\eta_s s''} \right) w_p(x, y) = 0 . \quad (4)$$

This expression is the modified Helmholtz equation that takes into account the behaviour of the plate coupled with the resonators. From Equation 4, it can be seen that the modified wavenumber is

$$k_{mod} = \sqrt[4]{k_b^4 + \frac{m_r'' \omega^2 s'' (1+i\eta_s) / B'}{m_r'' (\omega_0^2 - \omega^2) + i\eta_s s''}}. \quad (5)$$

2.3 The wall impedance

If the wall is a thin plate like in the case studied in this paper, the Kirchhoff plate equation describes the wave motion in the plate [22]. The wall impedance operator can be written as

$$\mathcal{Z} = \frac{B'}{i\omega} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 + i\omega m_p'' \quad (6)$$

The case presented in this study takes into account the influence of the resonators, so Equation 6 cannot be implemented directly. Re-writing Equation 4 it is possible to get an expression of similar form as

$$\mathcal{Z} = \frac{B'}{i\omega} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 - \left(\frac{m_r'' \omega s'' (1+i\eta_s)}{im_r'' (\omega_0^2 - \omega^2) - \eta_s s''} + \frac{B'}{i\omega} k_b^4 \right). \quad (7)$$

Substituting the bending wavenumber for the plate $k_b = \sqrt[4]{\omega^2 m_p'' / B'}$ into Equation 7 gives

$$\mathcal{Z} = \frac{B'}{i\omega} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 + i\omega m_p'' - \left(\frac{m_r'' \omega s'' (1+i\eta_s)}{im_r'' (\omega_0^2 - \omega^2) - \eta_s s''} \right), \quad (8)$$

which can be interpreted as the impedance operator of the plate plus a term controlled by the resonators. Assuming that the travelling wave is in the form of $e^{-i(k_x X + k_y Y)}$, k_x and k_y being wavenumbers, the wall impedance is reduced to

$$\mathcal{Z} = \frac{B'}{i\omega} (k_x^2 + k_y^2)^2 + i\omega m_p'' - \left(\frac{m_r'' \omega s'' (1+i\eta_s)}{im_r'' (\omega_0^2 - \omega^2) - \eta_s s''} \right). \quad (9)$$

In order to include the losses in the plate, the loss factor η_p is included as an imaginary part of the Young's modulus $E \rightarrow E(1 + i\eta_p)$ and by that also to the bending stiffness $B' \rightarrow B'(1 + i\eta_p)$.

2.4 Radiation Impedance

The formula of the radiation impedance of a finite plate utilized in this paper was derived and explained in a previous study [16], so here it is only presented for clarity

$$\begin{aligned} \mathcal{Z}_f &= \frac{ik}{2\pi S} \int_0^a \int_0^b 4 \cos(k\mu_x \kappa) \cos(k\mu_y \varrho) \frac{e^{-ik\sqrt{k^2 + \varrho^2}}}{\sqrt{k^2 + \varrho^2}} \\ &\times (a - \kappa)(b - \varrho) d\kappa d\varrho, \end{aligned} \quad (10)$$

where a and b are the dimensions of the plate in the x and y direction, S is the area of the plate ($S=ab$), k is the wavenumber on air, $\mu_x = \sin(\theta) \cos(\varphi)$, and $\mu_y = \sin(\theta) \sin(\varphi)$, θ is the evaluation angle and φ is the azimuth angle.

2.5 Effective mass

It is of interest to develop an expression of the effective mass (also referred to as apparent mass) of the proposed model. It is straightforward to do so from the wall impedance

$$m''_{eff} = B'(k_x^2 + k_y^2)^2 - \omega^2 m_p'' - \left(\frac{i m_r'' \omega^2 s'' (1+i\eta_s)}{i m_r'' (\omega_0^2 - \omega^2) - \eta_s s''} \right). \quad (11)$$

An approximate expression can be found if losses in the spring are neglected

$$m''_{eff} \approx \left(m_p'' + \frac{s''}{(\omega_0^2 - \omega^2)} \right). \quad (12)$$

In this form it is much easier to understand the behaviour of the effective mass. For low frequencies the mass of the plate and resonators are added. This means that, in this frequency region, the proposed model is effectively working as a wall with mass equal to the sum of the plate and resonators.

2.6 Theoretical band gap

It is known from the literature that this kind of arrangement, referred to as metamaterials, produce a frequency band where free wave propagation is not possible. Near this band gap is where sound transmission will be at its lowest [8-13, 18-20], so being able to predict or tune this band gap to a particular frequency range is of interest. It is important to understand which parameters play a role and how they influence this phenomenon. In order to prevent wave propagation, the wavenumber has to be imaginary. Analysing Equation 5 it is understood that in order for this to happen

$$k_b^4 + \frac{m_r'' \omega^2 s'' (1+i\eta_s) / B'}{m_r'' (\omega_0^2 - \omega^2) + i\eta_s s''} < 0. \quad (13)$$

If losses are not considered and after some modifications, this reduces to

$$\omega^2 m_p'' + \frac{\omega^2 s''}{(\omega_0^2 - \omega^2)} < 0. \quad (14)$$

The first condition is $\omega > \omega_0$ as this is the only way a term in Equation 14 could become negative to fulfil the presented inequality. Continuing with the derivation the upper frequency limit can be found. In Equation 15 the theoretical band gap is presented:

$$\begin{cases} \omega > \omega_0 \\ \omega < \sqrt{\omega_0^2 + \frac{s''}{m_p''}} \end{cases} \quad (15)$$

It is shown that the upper limit of the band gap is also related to the natural frequency of the resonators. The mass of the plate is an important factor in the band gap. From Equation 15 the size of the band gap can be expressed as

$$\Delta\omega_{BG} = \sqrt{\omega_0^2 + \frac{s''}{m_p''}} - \omega_0. \quad (16)$$

Substituting Equation 3 in Equation 16, $s = \omega_0^2 m_r''$ and defining a mass ratio $M = \frac{m_r''}{m_p}$ gives

$$\Delta\omega_{BG} = \omega_0(\sqrt{1+M} - 1). \quad (17)$$

As a result of this it can be stated that the frequency width of the band gap grows with mass ratio M . This is an important relationship to take into account when designing this type of construction.

2.6 Diffuse field transmission

The same approach utilized in Brunskog's paper [16] will be used in this study. For the sake of simplicity, the derivation of the equations will not be repeated here. The diffuse field transmission using Paris formula reads

$$\tau = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{4\rho^2 c^2 \Re\{z_f\}}{|z+2\rho c z_f|^2} \sin \theta \, d\theta d\varphi, \quad (18)$$

where ρ is the density of air, c is the speed of sound in air, θ is the incidence angle, φ is the azimuth angle, z_f is the radiation impedance and z is the wall impedance, equation 10 and 9 respectively in this paper. It is assumed that the resonators do not contribute to the sound radiation. In the following sections transmission loss (TL) will be used to analyse the proposed model. It is defined as

$$R = 10 \log \frac{1}{\tau}. \quad (19)$$

3. RESULTS AND ANALYSIS

Two test cases will be presented in this section to analyse the analytical model and the sound insulation behaviour of the proposed structure. In both cases a brick wall with dimension $2 \times 3 \times 0.05$ m is used. Material properties are shown in Table 1.

Table 1: Material properties

Property	Value
Young's modulus	17×10^9 [Pa]
Density	2000 [Kg./m ³]
Poisson's ratio	0.2

The stiffness of the springs for both cases are selected specifically to match the resonant frequency of the resonators to the coincidence frequency of the plate. As known from the equations presented and previous studies [8-13], the TL is at its highest in the resonant frequency. So tuning the resonators to the coincidence frequency, where the plate has the lowest TL, is the chosen method to counter act the coincidence effect problem of traditional wall constructions. The mass ratio M is one for both cases.

3.1 Without losses

The first case to be considered is a brick wall with the aforementioned dimensions and properties without considering the structural losses of the plate or the springs. Results are presented in Figure 2.

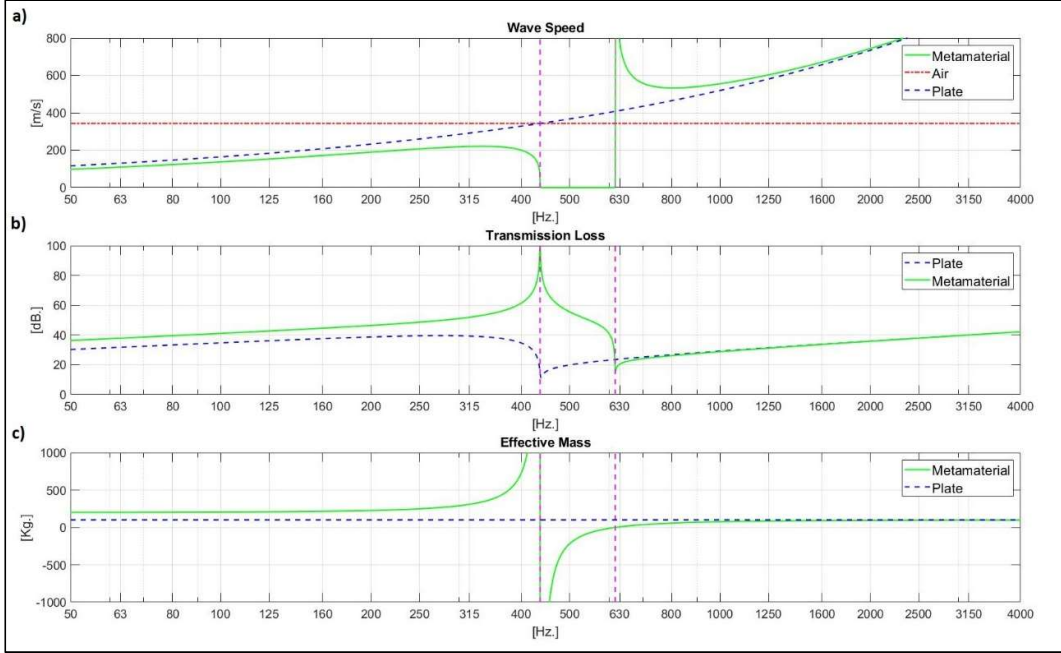


Figure 2: Results of the test case with no losses a) wave speed, b) transmission loss, and c) effective mass. Vertical dashed line indicates BG limits

Results include three graphs depicting (a) wave speed, (b) transmission loss, and (c) effective mass. In each one the metamaterial is compared to a plate without resonators. The theoretical band gap is indicated by the vertical dashed line. As expected, when losses in the springs are not being considered the approximation for the limits of the BG given in equation 18 become an exact solution. The wavenumber in this region takes an imaginary value so wave propagation is not possible.

Moreover, it can be seen that the transmission loss of the metamaterial has its maximum in the coincidence frequency of the plate, as it was designed. Below this frequency the metamaterial exhibits greater TL than the plate. It is exactly 6 dB higher, explained in the fact that the metamaterial has twice the mass of the plate ($M=1$). The displacement of the plate and the mass of the resonators are in phase below the resonant frequency and out of phase for higher frequencies. The effect of this can be seen in the fact that for frequencies above the BG the metamaterial and the plate have the same TL, even though the metamaterial has twice the mass. The coincidence effect problem is not solved by this proposed structure, but in fact shifted to a higher frequency as seen in figure 2(b). The TL dip is still present in the upper limit of the band gap.

The effective mass grows higher when approaching the resonance frequency, which provides the basis for the increment in transmission loss. Within the band gap the apparent mass is negative, converging to the mass of the plate for higher frequencies.

3.2 With losses

The next case is considering losses in the springs and the plate. Losses are $\eta=0.03$ for both. Results are presented in Figure 3.

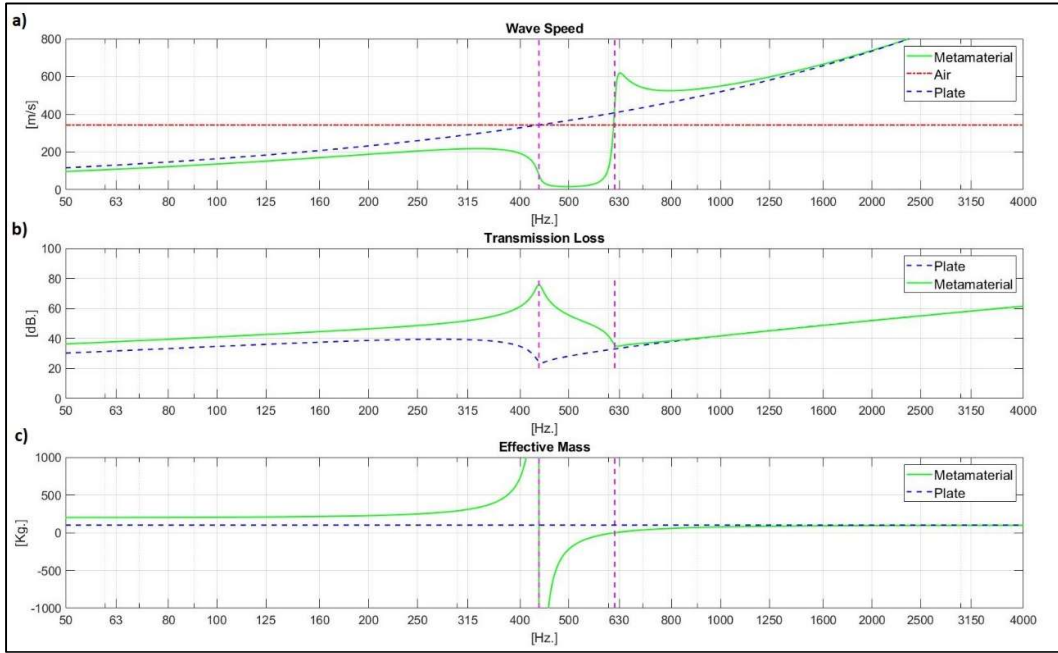


Figure 3: Results for the case considering structural losses of the plate and springs a) wave speed, b) transmission loss, and c) effective mass. Vertical dashed line indicates BG limits

When losses in the springs are being considered the theoretical band gap is no longer an exact solution, but still a good approximation. The presence of losses makes the width of the BG smaller, meaning that the higher the losses the smaller the band gap.

The peak of the TL is less prominent, but also the coincidence dip is softened. As expected the general behaviour of the structure remains the same.

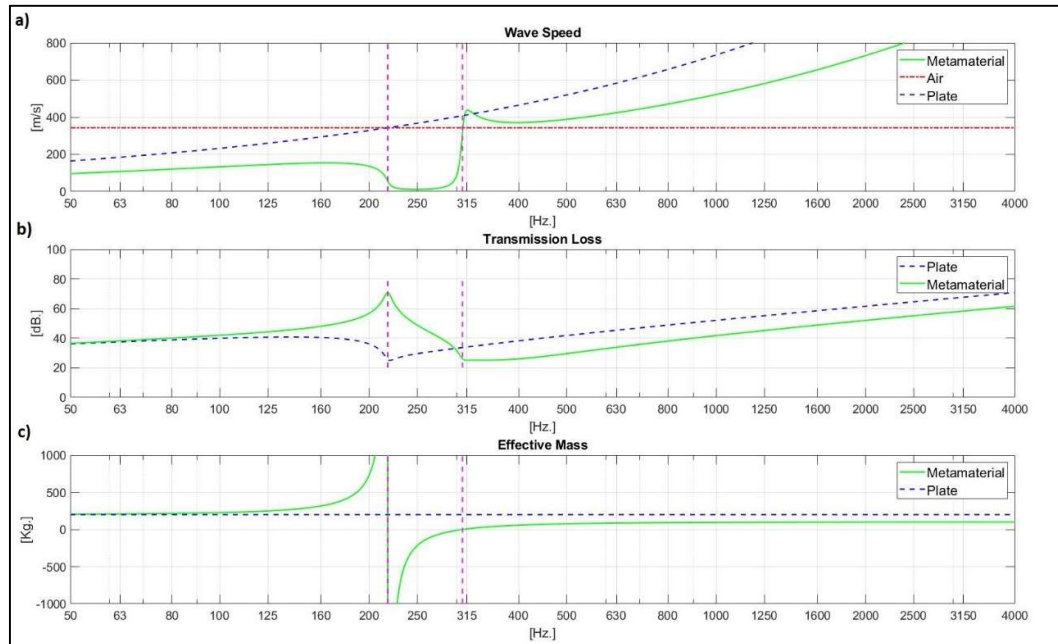


Figure 4: Results considering structural losses of the plate and springs compared to a plate of same mass a) wave speed, b) transmission loss, and c) effective mass. Vertical dashed line indicates BG limits

In figure 4 results of the same case are compared to a plate with equivalent mass, so as to better gauge the benefits of the proposed metamaterial in terms of transmission loss. This means that the plate has double the thickness because is modelled of the same material (0.1 m). As a consequence the coincidence frequency is lower. This seems to be a more fair comparison and the limitations of the model become evident. For frequencies above the band gap the TL of the metamaterial is much lower than the one of just the plate. So there seems to be a trade-off between the two approaches.

4. PARAMETER OPTIMIZATION

Knowing the limitations of the proposed structure brings the question of whether it is possible to optimize the parameters as to maximize transmission loss. In other words, to find a combination of mass ratio, spring loss factor, and spring stiffness that would provide the best possible sound insulation. However, defining what is better in terms of sound insulation is not a trivial task. To begin with, it would depend on the usage this structure is given. It can be used as a partition separating two rooms as it has been exemplified in the previous section but it also could be used for sound insulation in, e.g, cars. The spectrum of noises and sounds varies from case-to-case, so finding an overall best solution is problematic with the given limitations of this structure. ISO standard 717 [20] defines single-number quantities for airborne sound insulation of building elements such as walls. This provides a solid base to compare different solutions, at least if considering building structures.

4.1 Cost function

The weighted sound reduction index R_w [20] is the selected value to judge which solution is better. The cost function used in this paper is

$$\varphi = \left(\frac{1}{R_w} \right)^2, \quad (20)$$

with R_w being the sound reduction index of the metamaterial solution. This means that R_w will be maximized.

4.2 Constrained Optimization test case

The developed cost function is used to optimize the case presented in section 3.2. In order to keep the solutions realistic upper and lower bounds were used. The constraints and optimized values are shown in table 2.

Table 2: Constraints used for optimization and optimized values

Property	Lower Bound	Upper Bound	Optimized value
M	0.1	1	0.97
η_s	0.001	0.5	0.29
s''	1×10^8	1×10^{10}	2.48×10^9

Different algorithms were tested. Gradient based optimization was not suitable for this case. The cost function is not sensitive enough to changes in the optimization parameters so the algorithm would quickly converge to a solution. The initial guess is basically the result. Taking this into account a genetic algorithm was used for the optimization (Matlab's "ga" function). In this way most permutations are tested, almost

ensuring a good result at the cost of efficiency. At this time, efficiency is not a concern so this is an acceptable drawback. Figure 5 presents the result of the optimization.

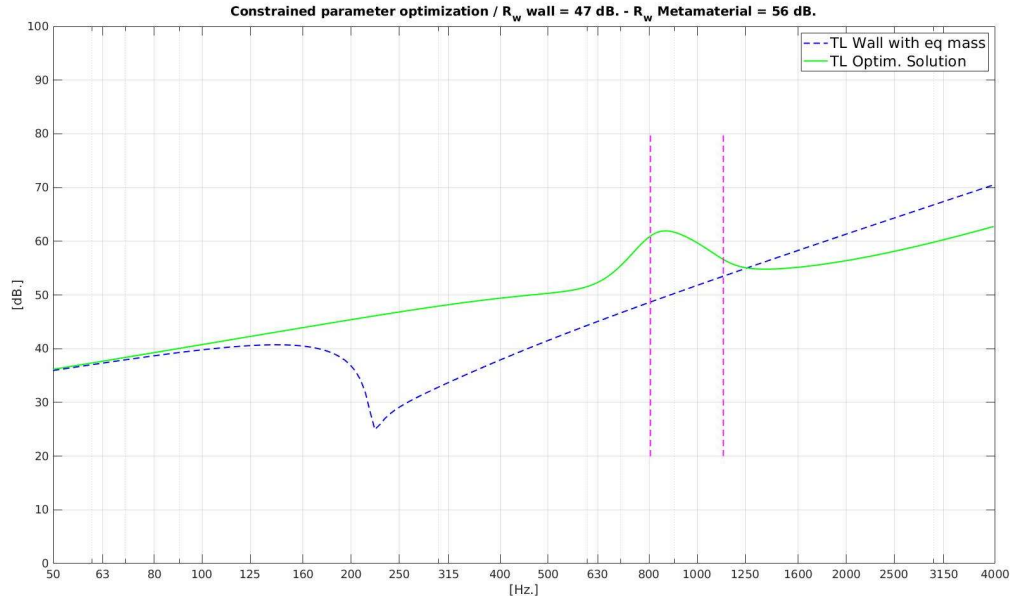


Figure 5: Constrained parameter optimization TL results. Vertical dashed line indicates BG limits

The weighted sound reduction index of the optimized metamaterial solution is $R_w=56$ dB meanwhile the value for the wall with equivalent mass is $R_w=47$ dB. Furthermore, the minimum TL value of the optimized structure is bigger than the minimum value of just the wall. The optimization process seems to have maximized the frequency range where the transmission loss of the metamaterial is larger than the wall and tuned the BG above the coincidence frequency. The coincidence effect is smoothed out by the losses in the springs. The feasibility of the optimized values shown in Table 2 are not explored in this study.

On the other hand, for frequencies above the band gap the TL of the wall is higher than the metamaterial.

5. DISCUSSION

The developed analytical model for sound transmission loss of a finite SLW with a coupled array of single degree of freedom resonators was tested and analysed. The behaviour of the proposed metamaterial structure is on par with experimental and numerical results of various studies [18-20]. In the frequency region around the natural resonance of the resonators, a range exist were wave propagation is not possible. The developed expression for this band gap confirms that the size of this region is dependent on the mass ratio and the resonant frequency. An increase in the mass of the resonators (or a decrease in the mass of the plate) leads to a larger band gap.

The transmission loss grows with frequency, reaching its maximum in the lower limit of the band gap (approximately the resonant frequency). This is followed by a decrease until it reaches its minimum value around the upper limit of the BG. This behaviour can be explained by looking at the effective mass. Towards the natural frequency of the resonators, it also grows to its largest value. Within the band gap, the effective mass becomes negative. For frequencies above the BG, the mass of the

resonators and the plate are moving out of phase, while below they are in phase. The developed model is restricted to the frequency range where the wavelengths of the vibrations travelling through the wall are much longer than the periodic distance between the resonators. This provides a limitation when designing these types of structures.

Furthermore, it is of interest to discuss the feasibility of the proposed structure as a tool to increase sound insulation at constant mass per unit area, paying special attention to the coincidence effect problem. Tuning the resonators to the coincidence frequency provides good TL rendering that problem void, but it also introduces a new higher coincidence frequency. This means that the problem is just shifted to another frequency. Added damping provides a way to minimize this new dip in TL but it still exists. The optimization process presented in section 4 shows a way to broaden the frequency region where the sound insulation of the metamaterial is superior to the one of the wall of equivalent mass. In this case, the BG was tuned above the coincidence frequency with springs with high losses. Being guided by the weighted sound reduction index proposed in ISO 717 one might conclude that the proposed structure is a superior sound insulator. For frequencies above the BG the metamaterial structure provides less sound insulation. In future work it would be interesting to test metamaterials based on multiple degrees of freedom resonators and study their effectiveness in combating the coincidence effect problem. It would also be of interest to validate this analytical model with numerical simulations and experimental data.

6. CONCLUSIONS

The analytical model developed in this paper is useful to better understand metamaterials composed of single degree of freedom resonators and how the different parameters affect their behaviour. The proposed structure is an effective sound insulator. The coincidence effect problem is not resolved but shifted to an upper frequency. The possibility of using springs with high losses and tuning the band gap above the coincidence frequency emerges from the optimization process as a possible solution to this phenomenon.

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