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To Infinity and Beyond - the Amazing Uses of Infinite Structure Mobility Theory

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ABSTRACT

What if there were simple formulae you could use to calibrate structural mobility measurements on beams, plates, pipes, large pressure vessels, aircraft fuselages, and other structures? What if those same formulae could be used to estimate the mobilities of structures that have not yet been built? How about using them to estimate how mobility might change if you modify the material properties of an existing structure? Great news - these formulae exist, are simple enough to code in a spreadsheet in minutes, and are perhaps the most invaluable tools a structural-acoustician has. They simulate propagating waves in infinite structures, including beams, plates, and curved shells. In this plenary lecture I will explain these formulae and prove their worth with several practical examples.

Keywords: Vibration, Mobility, Infinite Structure
I-INCE Classification of Subject Number: 42
(see http://i-ince.org/files/data/classification.pdf)

1. INTRODUCTION

It is sometime in the early 1990s and I am a young engineer struggling to compute the forced vibration response of a complicated structure. My finite element model is only accurate up to a few hundred Hz and can't be refined further since I have maximized the available memory on the supercomputer I am using. My sponsor expects analysis results at much higher frequencies soon and I am out of options. One day I complain to an older colleague over lunch and he asks a few questions about my structure. "How thick is the plating and what is it made of?" I tell him, and then watch him write a short formula and calculation on the back of envelope (see the reproduction in Figure 1). "Try overlaying this value on your calculations. If it's close at higher frequencies, you may be able to use it as a pretty good estimate."

I am skeptical. "This is a complex structure. How can a simple formula estimate its response?" My colleague is confident, though, "At high frequencies, there are enough

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modes and damping for you to use infinite plate mobility theory." My skepticism grows, "This structure is finite, not infinite," I insist, and point at the envelope, "besides, where is the frequency dependence in this formula?" My friend smiles. "There is none, it works at all frequencies. Just try it."

I return to my office, pull out one of my mobility plots (still skeptical), and draw a horizontal line with the value my colleague calculated along the upper frequency range (see the reproduction in Figure 2). I am amazed as the value indeed bisects the calculations I had spent the better part of a month performing on the supercomputer. Not only that, but the deviations in amplitude between the finite and infinite mobility are quite small. I can truly simply estimate the higher frequency behavior of my complicated structure using the infinite plate formula and an added variability of a few dB.

The next day at lunch I thank my older colleague, and ask him to explain the theory. I am a convert - I now believe in the usefulness of the infinite. You will also after reading this tutorial.

This paper continues a series on vibro-acoustics tutorials initiated in Acoustics Today magazine [1, 2], and continued at Internoise conferences [3–5]. I therefore omit details of basic vibroacoustic analyses in this paper, referring readers to the previous articles. You can download them at www.hambricacoustics.com. Also, if any of the animated figures in this PDF aren't working, you can also download a new version at that website.

Finally, I take no credit for the infinite structure mobility equations provided here. Instead I extend my respectful and deep thanks to Drs. Cremer, Heckl, and Ungar [6] and Dr. Skudrzyk [7] who championed the use of simple theories to approximate the response of complex structures. Their methods have truly stood the test of time and provide great value to vibroacoustians around the world today. In this paper, I hope to remind the community of the usefulness of these formulae, and point out some Amazing Uses which are not commonly known.

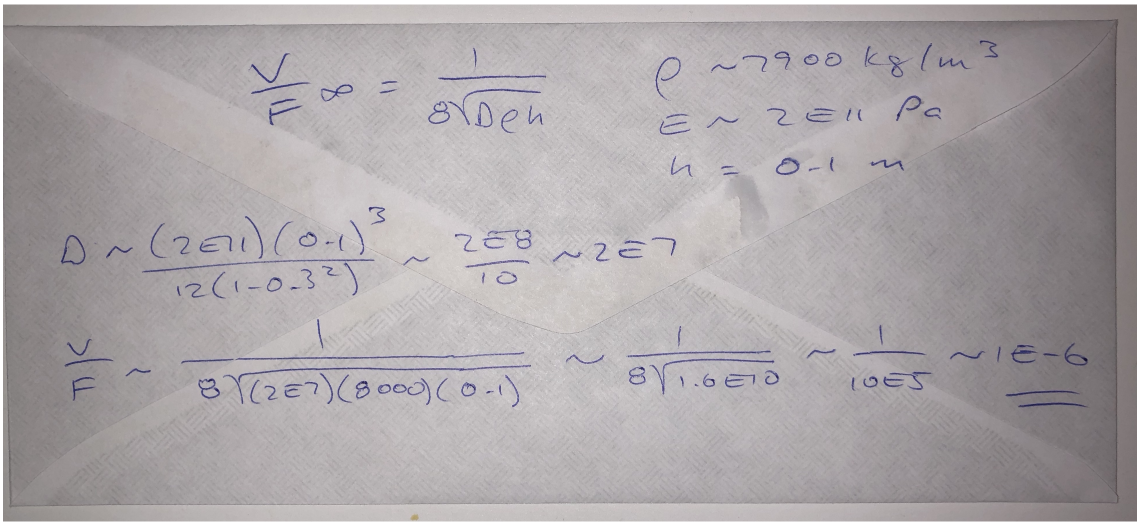

Figure 1: Recreation of back of the envelope infinite panel mobility calculations.

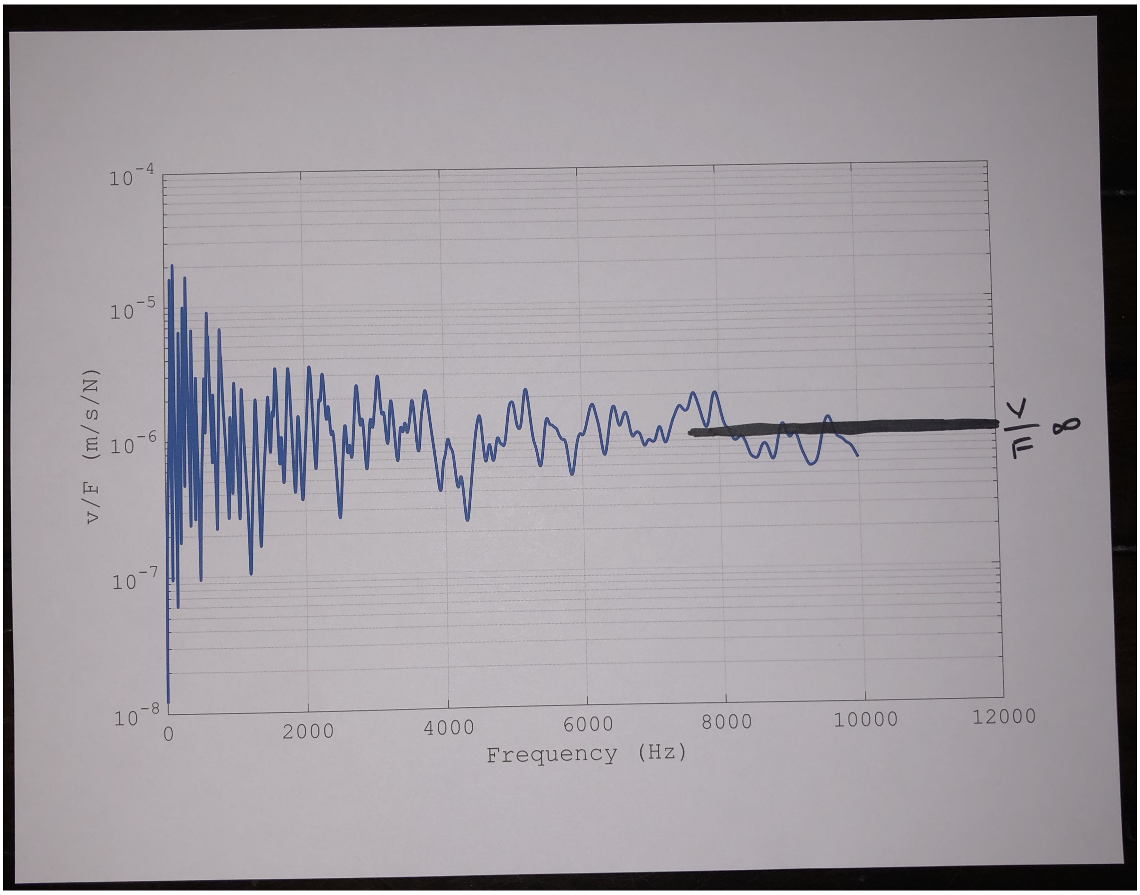

Figure 2: Recreation of circa 1990s mobility calculation with superimposed infinite panel high frequency estimate.

2. INFINITE AND FINITE WAVEFIELDS AND CHARACTERISTIC IMPEDANCES

Watch the animated flat panel waveforms in Figure 3. In this example I have added a right traveling wave (top) to a reflected left traveling wave (middle) to produce a standing wave (bottom). The left side of the wave represents a drive location and the right side a free edge. The incident wave propagates to the right until it strikes the free edge. The reflected wave travels back towards the drive, in this case constructively interfering with the incident wave to produce a standing wave pattern, like what occurs in a mode shape. Notice that the amplitude of the standing wave is twice that of the left and right traveling waves.

Now consider the animated waveforms in Figure 4. In this case I have added uniform damping to the panel, so that the left and right traveling waves lose energy as they propagate. In this example the loss factor is so high that the amplitude of the reflected wave which returns to the drive is very small. Now, the summed wave pattern is not so different from the original right traveling wave, particularly near the drive point. This is the basis of infinite structure theory - that reflected waves in large damped structures are small with respect to incident waves.

The mobility of infinite waves, called the 'characteristic mobility' by Skudrzyk, is analogous to that in acoustic fluid media. The particle velocity induced by a time harmonic fluctuating pressure in an infinite fluid is inversely proportional to the

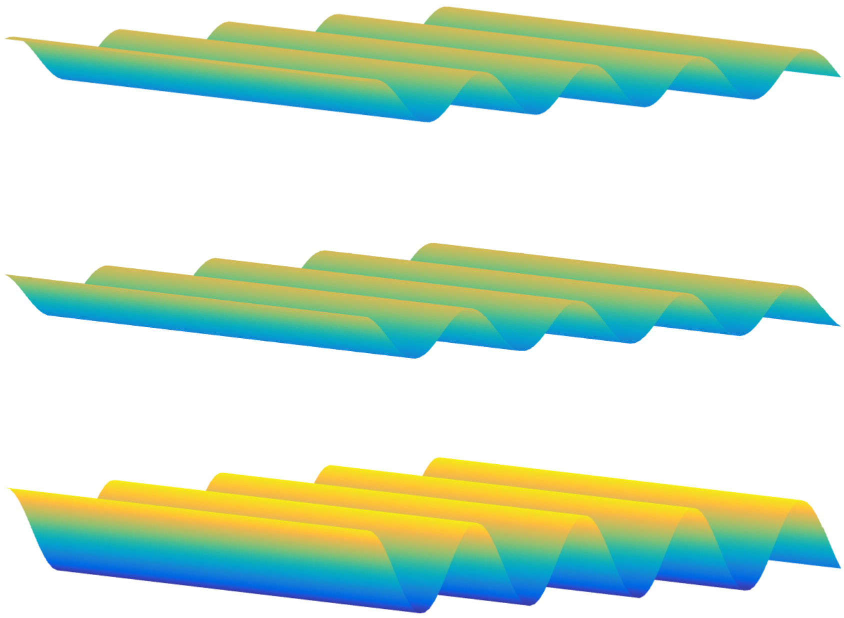

Figure 3: Propagating waves in panel - undamped; top - incident wave, middle - reflected wave, bottom - sum of waves

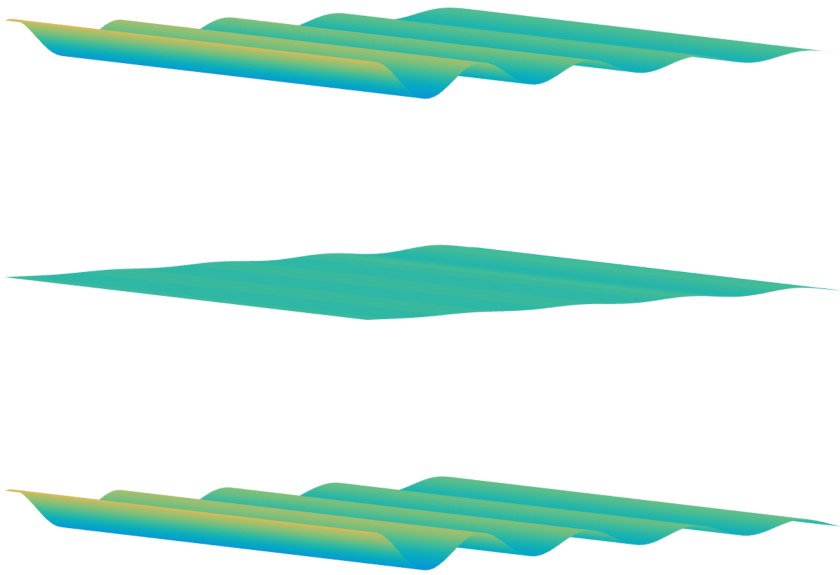

Figure 4: Propagating waves in panel - highly damped; top - incident wave, middle - reflected wave, bottom - sum of waves

'characteristic impedance' $\rho_o c_o$:

$$(v/p)_{char} = \frac{1}{\rho_o c_o} \tag{1}$$

where ρ_o is the fluid mass density and c_o is the fluid speed of sound.

The characteristic mobility of a compressional wave in an infinite beam or rod is the same, but with the structural density and sound speed, along with an area term to replace the applied pressure with an applied force:

$$(v/F)_{inf} = \frac{1}{\rho_s A c_l} \tag{2}$$

where $\rho_s A$ is the structural mass density per unit length and $c_l = \sqrt{E/\rho_s}$ is the wavespeed of longitudinal waves in the structure where E is Young's Modulus of elasticity. The same formula applies for torsional waves, but with c_l replaced by the shear wave speed $c_s = \sqrt{KG/\rho_s}$ where K is the fraction of the beam cross sectional area which contributes to shear stiffness.

The mobility for flexural waves in an infinite beam is nearly identical, but with some additional factors. When driving transversely the center of an infinite beam the infinite panel mobility is

$$(v/F)_{inf} = \frac{1}{2\rho_s A c_b} \tag{3}$$

but when driving the free end of a $semi-infinite$ beam, the mobility is four times higher (since there is no shear and moment resistance from the 'missing' half of the beam):

$$(v/F)_{inf} = \frac{2}{\rho_s A c_b} \tag{4}$$

For a thin beam, $c_b = \sqrt[4]{\frac{EI}{\rho_s A}\omega^2}$. Since $c_b \propto \sqrt{\omega}$, the infinite beam mobility decreases with the square root of increasing frequency. We will see examples of this later in the paper.

3. FINITE AND INFINITE FLAT PANEL MOBILITIES

Bending waves in a two dimensional plate behave similarly, but since the waves propagate cylindrically away from the drive the mobility equations are a bit more involved. There are different ways to derive the characteristic mobility for plates. Here, we'll use the concept of a transverse point drive inducing plate deformation which resembles a circular piston. We can equate the panel shear response around the piston circumference to the drive force, and then write an equation relating the panel shear response to that of the bending waves. We combine the equations to produce the general infinite flat panel mobility (see the schematic and equations in Figure 5 for details):

$$(v/F)_{inf} = \frac{\omega}{8Dk_b^2} \tag{5}$$

Next, we use the relation $k_b = \omega/c_b$ along with the thin panel bending wavespeed

$$c_{bthin} = \sqrt[4]{\frac{D\omega^2}{\rho_s h}} \tag{6}$$

to produce the equation written on the envelope in Figure 1 :

$$(v/F)_{inf} = \frac{1}{8\sqrt{D\rho_s h}} \tag{7}$$

where D is the flexural rigidity $Eh^3/(12[1-\nu^2])$ and h is the panel thickness.

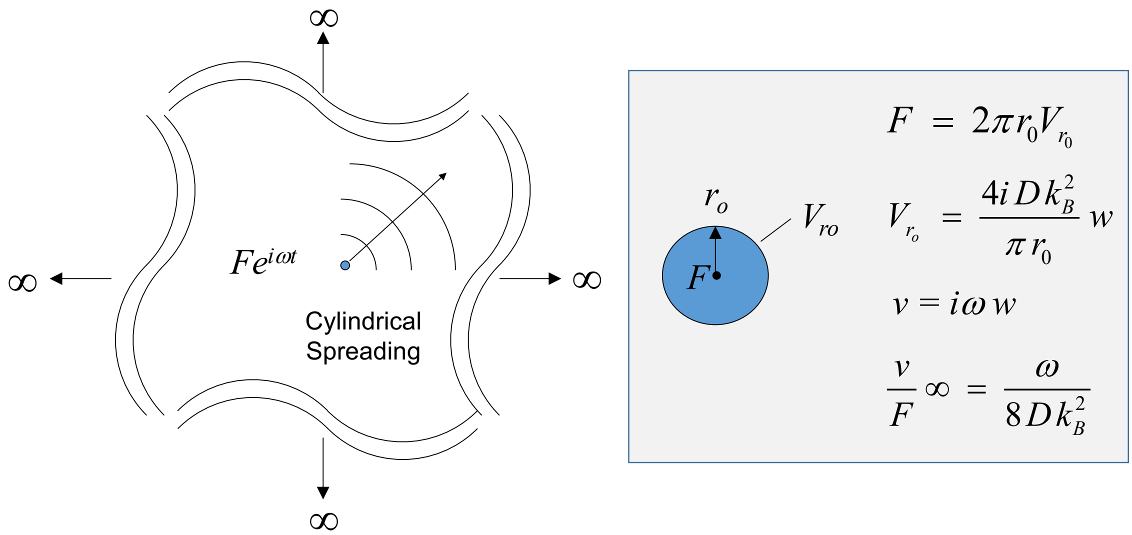

Figure 5: Derivation of infinite flat panel mobility.

Now let's compare the thin infinite panel flexural wave mobility to that of a finite panel. See [1] for the formulation of a finite panel flexural mobility as the sum of its modes. Recall from the examples in Figures 1 and 2 that these modes are just superpositions of waves reflecting from the panel boundaries (animated examples are in [5]). Figure 6 shows the mobility of a rectangular simply supported finite panel (including each of the individual modal responses in the summation) along with the thin infinite panel mobility of a plate with thickness and material properties shown in the figure (this is a Matlab Graphical User Interface I use for teaching Sound-Structure Interaction at Penn State which allows students to adjust parameters to examine the effects on mobility). Here we can see our first Amazing Use:

Amazing Use 1: Infinite structure mobility represents the geometric mean of a finite structure mobility

This observation is made often by Skudrzyk, and simply means that the characteristic mobility of an infinite wave falls between the maxima and minima of constructively and destructively interfering wave patterns. Note that the geometric mean is the square root of the product of the upper and lower mobility envelopes. When mobility is plotted on a log scale, the infinite panel mobility appears to be halfway between the upper and lower finite mobility bounds. This thin flat panel infinite flexural wave mobility does not depend on panel size or boundary conditions. It is also frequency dependent for flat thin panels, and real-valued (as infinitely propagating waves are purely resistive) making it extremely handy for some other amazing uses.

As structural damping increases, the finite panel mobility is less affected by reflected waves (recall the waves in Figure 2). Figure 7 animates the mobility from Figure 6 over increasing loss factor, showing decreasing amplitude variability. As loss factor increases, the next Amazing Use becomes apparent:

Amazing Use 2: Infinite structure mobility represents the high frequency finite structure mobility to within a few dB

Of course, this is what inspired me to investigate infinite structure theory after the revelations described in the introduction and shown in Figure 2.

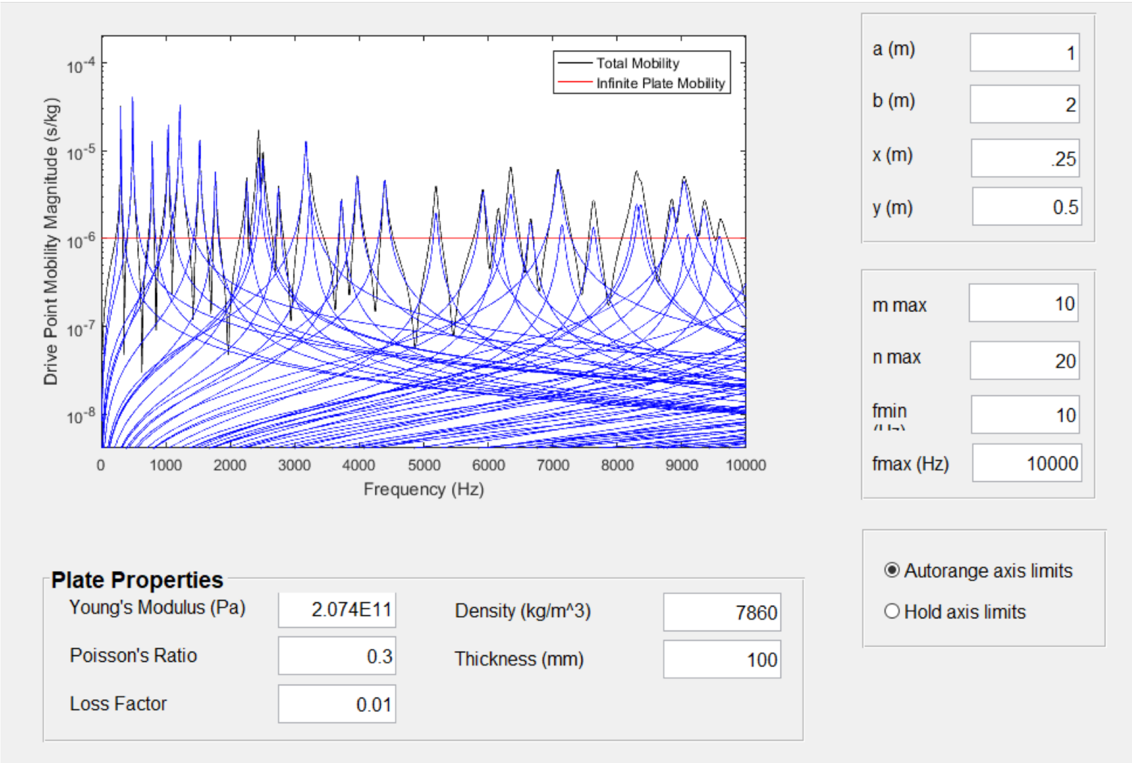

Figure 6: Flat panel mobility, loss factor of 0.01. The blue curves are mobilities of individual modes, the black curve is the total finite panel mobility, and the red curve is the infinite panel mobility.

Another Amazing Use is less apparent. During my mentoring of graduate students and young engineers, I have had them measure the mobilities and modes of vibration of simple and complex structures. Many of the mobility plots I have been presented had obvious errors, sometimes due to inverted calibration factors (Engineering Units/Volt instead of Volts/Engineering Unit), forgotten gains on various channels, incorrect units (acceleration instead of velocity, acceleration in g's instead of Engineering Units), and mixed units (m/s/lbf instead of m/s/N). Here then is another Amazing Use:

Amazing Use 3: Infinite structure mobility may be used to check the quality of measured mobilities

Figure 8 shows some examples of erroneous measurements (simulated using analytic modal summations for this tutorial), along with the correct one. This amazing use is not

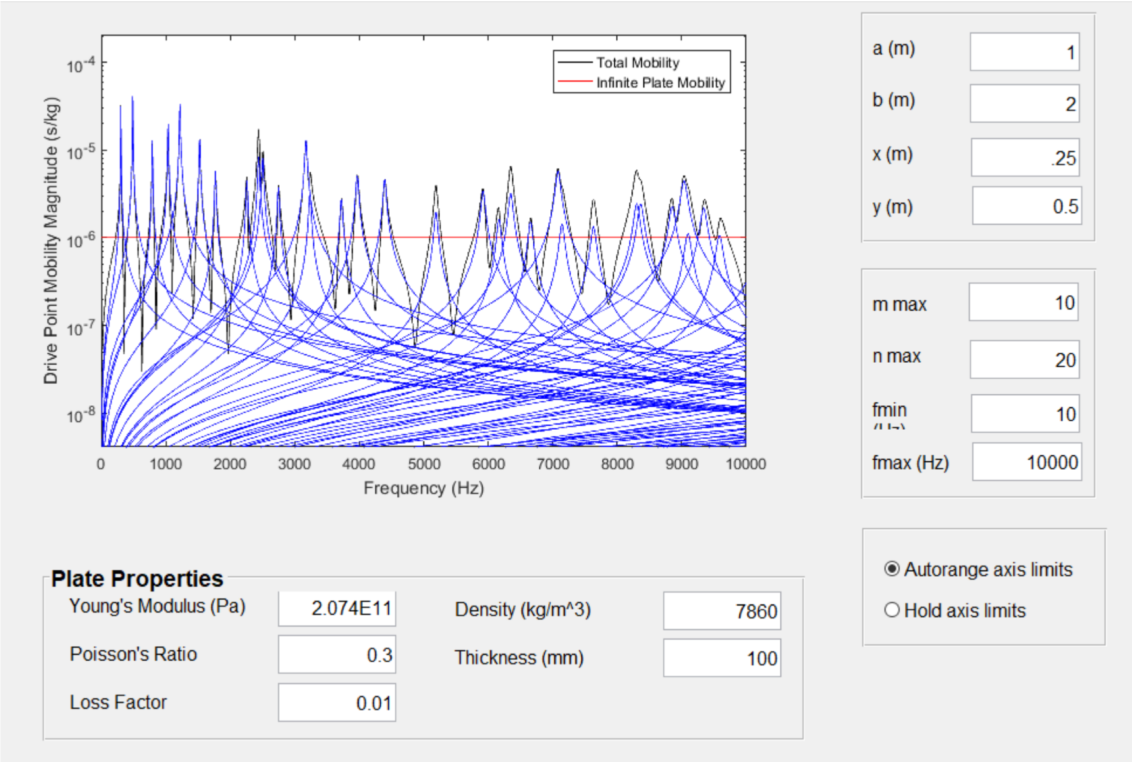

Figure 7: Flat panel mobility variations with structural damping

unique to flexural waves in thin panels, as we'll see in examples of other infinite structure mobility formulae shortly.

You can also estimate the effects of material changes on structural mobilities using infinite structure theory. Consider the ribbed panel shown in Figure 9. Two panels were machined, one from Lexan (a plastic) and another from Aluminum (there are no fasteners in either panel). Typical mode shapes of the ribbed panel, which correspond to different resonance frequencies for the two materials, are also shown in the Figure. The top mode shape is a *global* mode, where the ribs both stiffen and mass-load the plate mode shape. The middle and bottom modes are *local* modes, with mode patterns 'trapped' between or outside the ribs.

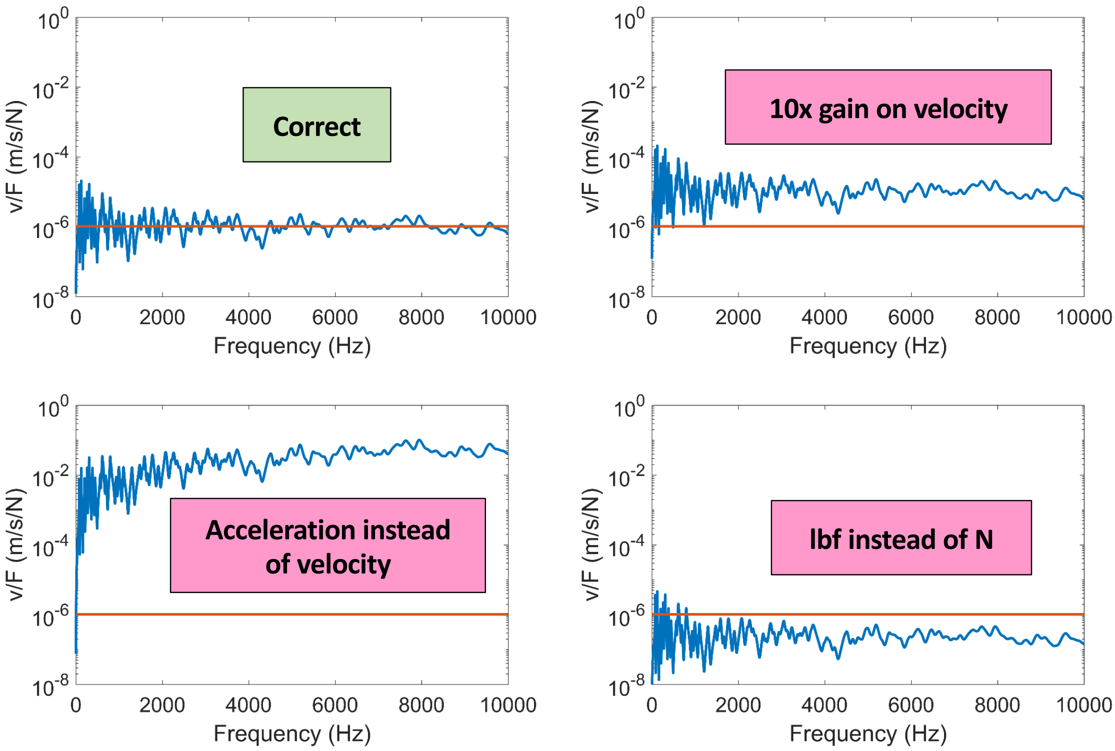

Figure 8: Examples of correct (upper left) and erroneous measured mobilities checked against infinite panel mobility.

Each panel was driven transversely at its corner to measure flexural mobility. The results, along with infinite panel mobility estimates, are shown in Figure 10. Note that the infinite panel mobilities were multiplied by 4 to account for the impedance reductions due to driving a corner. While not an exact correction, it is suitable to demonstrate the next Amazing Use:

Amazing Use 4: Infinite structure mobility may be used to scale the mobilities of geometrically identical structures made of different materials

Figure 11 shows the mobilities again, but now with the Aluminum panel mobility scaled to that of the Lexan panel. There are actually two scaling steps: first, the mobility amplitude is scaled by the ratio of the Aluminum and Lexan infinite panel mobilities, and second, the frequency axis of the Aluminum mobility is scaled by the ratio of the material longitudinal wavespeeds $\sqrt{E/\rho}$:

$$\frac{f_{Al}}{f_{Lex}} = \frac{c_{Al}}{c_{Lex}} = \frac{\sqrt{E_{Al}/\rho_{Al}}}{\sqrt{E_{Lex}/\rho_{Lex}}} \tag{8}$$

The scaled mobilities overlay fairly well, with some differences. First, the resonant and antiresonant peaks are sharper for the Aluminum panel, due to its lower material damping (this is most evident in the local modes with the smaller peaks). Second, some of the low frequency resonance peaks are misaligned. This is due to slight 'bowing' of the Aluminum panel, caused by the machining process. This bowing couples membrane

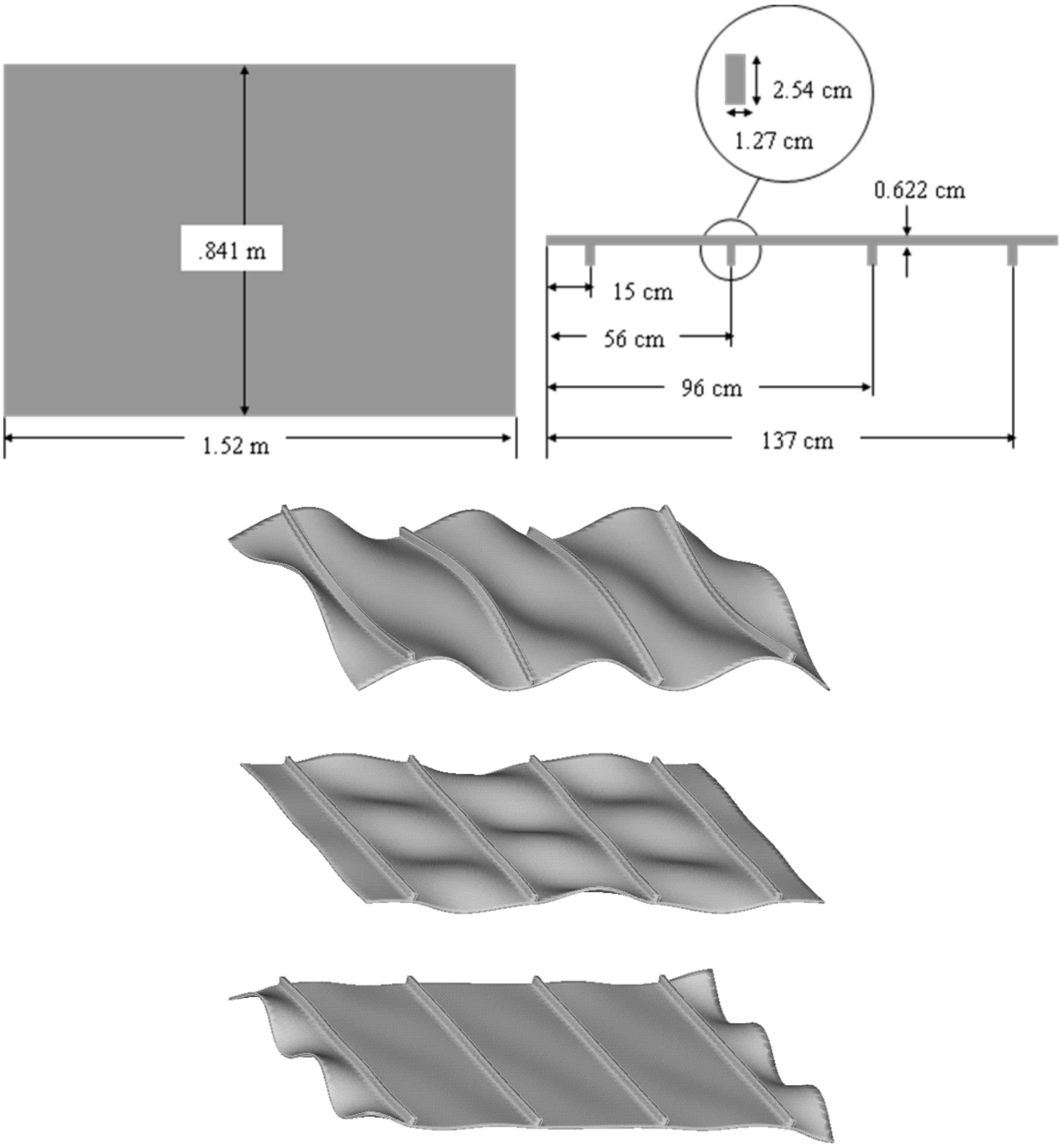

Figure 9: Top: Ribbed panel manufactured from Aluminum and Lexan. Bottom: examples of global and local panel mode shapes

waves to the flexural ones, stiffening the mode shapes and therefore the resonance frequencies. We will discuss this membrane-bending coupling further when we consider infinite waves in cylindrical shells.

While an interesting example, consider the more important ramifications of Amazing Use 4. Suppose you are asked to assess the potential vibro-acoustic impact of changing the material of an existing structure, while not changing its geometry. Do you need to rerun large computational models to assess this effect? Or, do you need to manufacture and test prototypes? Neither! Simply use the procedure shown here to scale an existing mobility to a new one for a different material (don't forget the frequency scaling). Furthermore, if there are certain vibration acceptance criteria, you can use infinite structure scaling to specify allowable material properties. For example, the modulus may not be decreased by more than a certain amount before a maximum allowable vibration level would be violated.

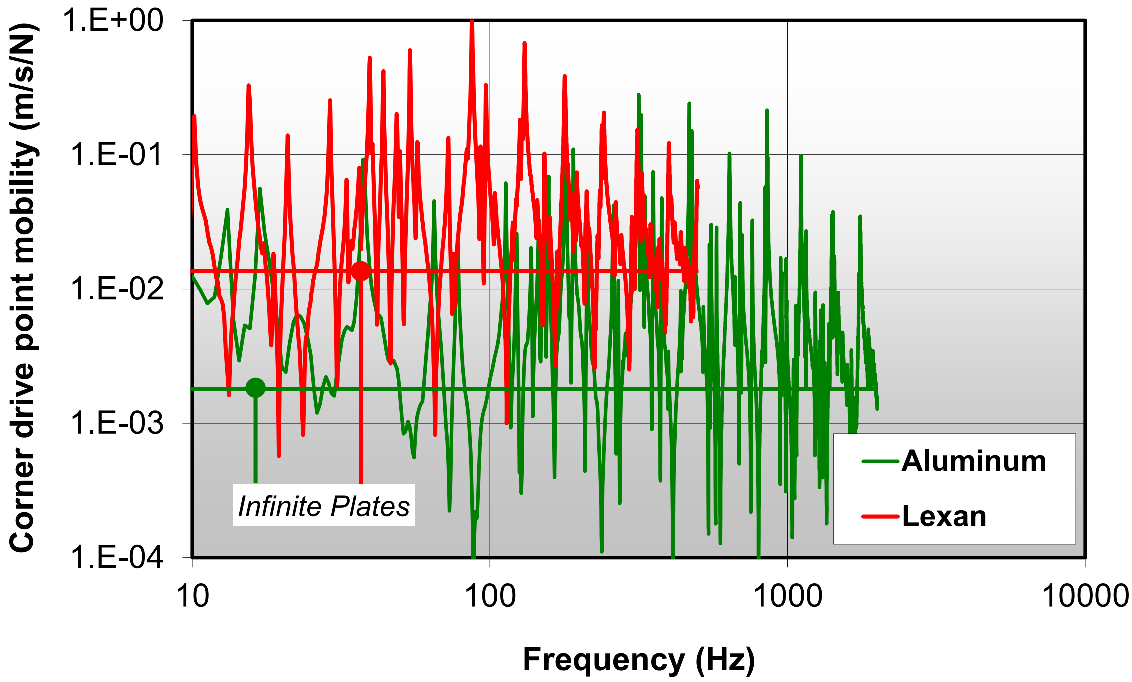

Figure 10: Corner drive point mobilities of the Aluminum and Lexan ribbed panels.

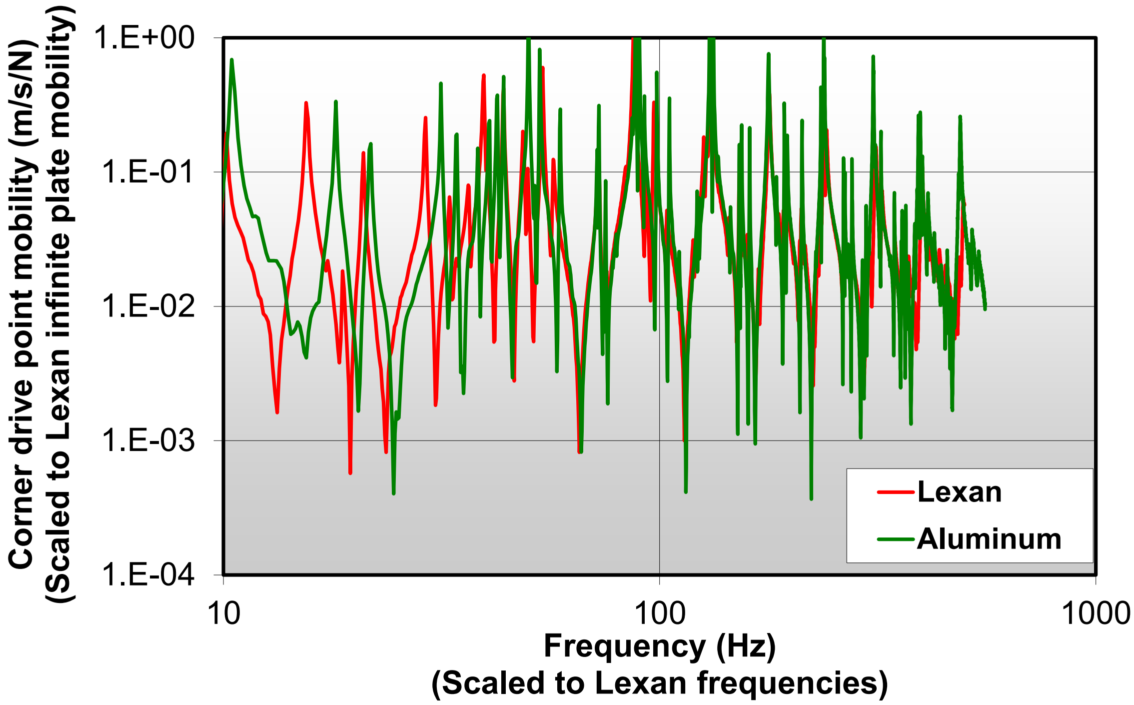

Figure 11: Corner drive point mobilities of the Aluminum and Lexan ribbed panels, scaled to Lexan frequencies and amplitudes.

4. SANDWICH PANELS

Many structural panels used in aerospace and building construction are *sandwich panels*. Thin, but stiff upper and lower face sheets are adhered to a lightweight core to produce a stiff, strong, lightweight panel. An example of sandwich panel construction typical of aerospace structures is shown in Figure 12. The core is Aluminum honeycomb and the facesheets are carbon fiber composite laminates. These types of panels are so stiff and light that simple thin panel mobility theory becomes invalid at mid to high frequencies. Ref. [4] provides a tutorial on sandwich panel wavespeeds, mobilities, modes of vibration, and sound radiation, which I will not repeat here.

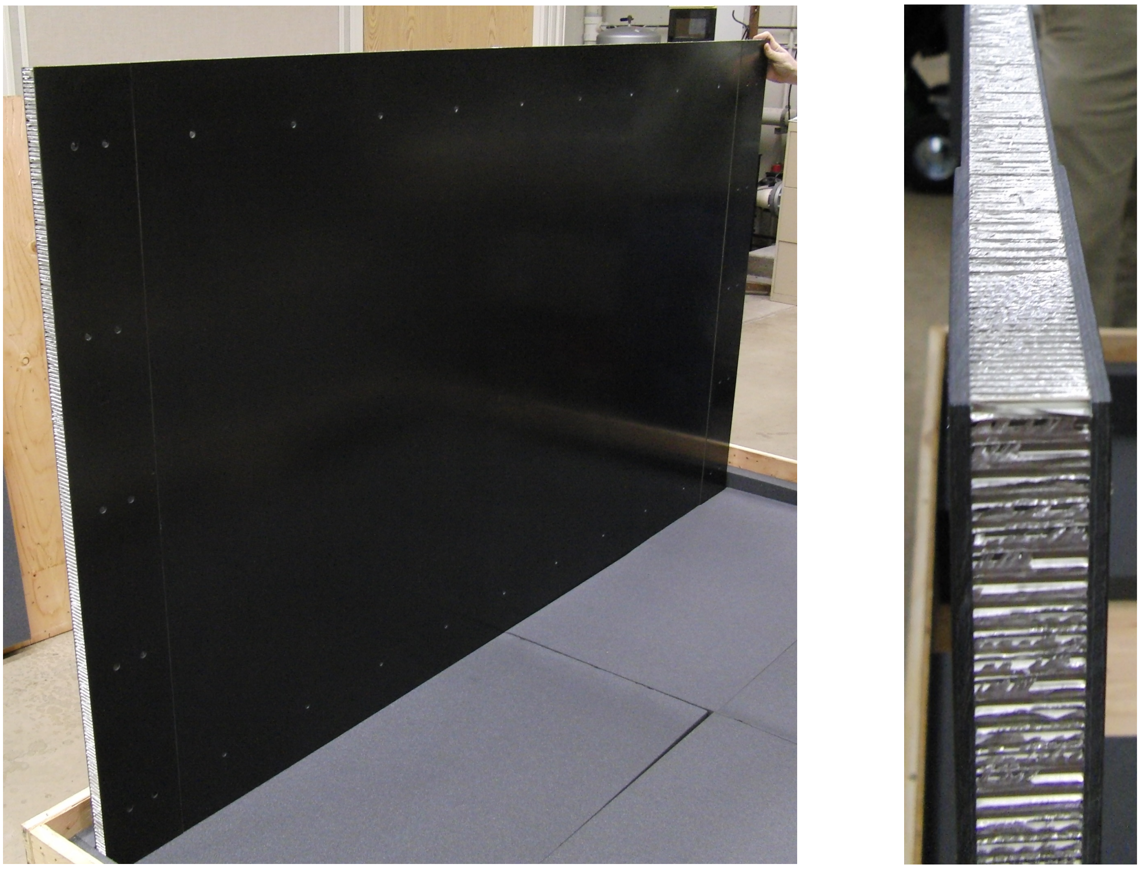

Figure 12: Sandwich panel with carbon fiber facesheets and a hexagonal aluminum honeycomb core.

I will, however, explain qualitatively how sandwich panel transverse waves vary with frequency. At low frequencies, the face sheet stiffness is dominant, and the waves look like the traditional bending waves shown in the top of Figure 13. As frequency increases, however, the core shear becomes dominant, and pure shear waves appear, as shown in the bottom of Figure 13. The wavespeed of the flexural waves depends on the face sheet thickness and material properties, but more importantly on the core thickness. The core offsets the face sheets from the neutral axis significantly, increasing their effective stiffness (again, for details see [4]). The shear wavespeed ($c_s = \sqrt{0.83G/\rho_s}$) is much lower than that of flexural waves. The mobility of an infinite sandwich panel therefore transitions between that of flexural (face sheets) to shear (core) waves with increasing frequency. A good approximation of this transition (avoiding some painful math) is:

$$(v/F)_{inf} = \frac{\omega}{4\mu_s c_b^2}\left(1 - \frac{1}{2}\left(\frac{c_b}{c_{b_{fs}}}\right)^3\right) \tag{9}$$

where c_b is the total panel wavespeed, $c_{b_{fs}}$ is the wavespeed considering only the facesheets, and μ_s is the total panel surface density[2]. Note that at low frequencies, $c_b = c_{b_{fs}}$ and the equation becomes identical to Equation 7 (after substituting the thin panel wavespeed equation). At high frequencies, $c_b \rightarrow c_s << c_{b_{fs}}$ and the mobility becomes that of a pure infinite shear wave.

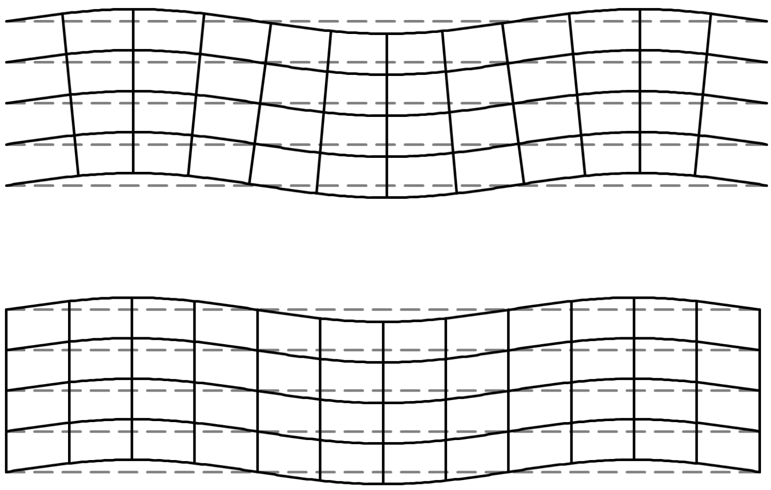

Figure 13: Flexural wave (top) and shear wave (bottom).

Figure 14 compares the surface averages[3] of two mobility measurements - one made using a roving force hammer and accelerometers, and the other using an electrodynamic shaker instrumented with a force gage and accelerometer. As was the case for thin panels, the infinite panel mobility for the sandwich panel is the geometric mean of the finite panel mobilities. The effects of the core shear become apparent in both the measured and infinite structure mobilities above 1 kHz, giving us:

Amazing Use 5: Sandwich panel infinite structure mobility estimates mean finite sandwich panel mobility at low and high frequencies

The slower (and lower stiffness) shear waves have a higher mobility, which increases with frequency, unlike the flexural wave mobility which is frequency independent. The infinite sandwich panel mobility can be used to easily identify the transition frequency between flexure and shear.

[2]The summation of the individual ρh values of the face sheets and core

[3]This is the average of many drive point mobility measurements made over the panel surface.

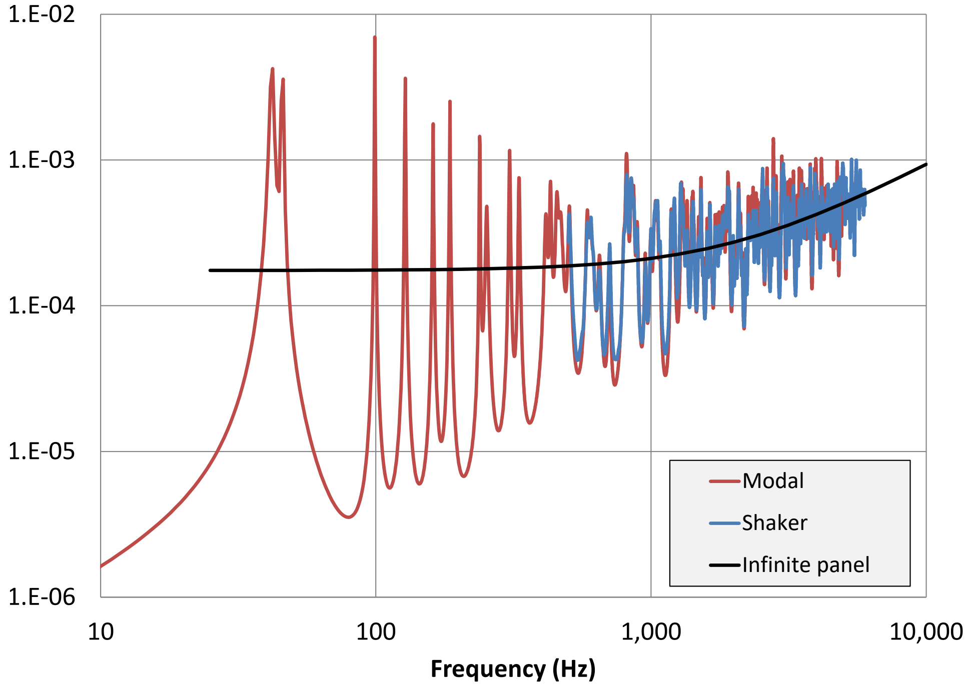

Figure 14: Measured and infinite sandwich panel mobilities.

5. CYLINDRICAL SHELLS

Consider now the cylindrical shells shown in Figure 15. Cylindrical shells are often defined by the geometric ratios L/a and h/a, where L is length, a is radius, and h is wall thickness. Shells with large L/a and h/a are essentially pipes, like the structure in the lower right of the figure. As we will see, these structures resemble and behave like beams (at low frequencies anyway). Shells with small h/a, like the ones shown on the left, are analogous to large pressure vessels. These structures behave like shells, with mobility based on both flexural and in-plane membrane behavior. The structure shown on the upper right is a small reciprocating compressor housing, courtesy of Bristol Compressors. The compressor internals are complex and fill most of the internal volume. There are also internal stiffeners, and the refrigerant is pressurized, preloading the shell walls. In spite of these complications, we will soon see that simple infinite shell theory represents the mean mobility of this structure quite well.

There are three equations for cylindrical shell mobility, each of which is based on another key cylindrical shell parameter: the *ring frequency*:

$$f_{ring} = \frac{c_l}{2\pi a} \tag{10}$$

where c_l is the longitudinal wave speed of the membrane waves in the shell and $2\pi a$ is the shell circumference. So, the ring frequency is just the frequency at which the wavelength of a longitudinal wave matches the circumference. This corresponds to *breathing* motion of the shell, and strong sound radiation. The ring frequency is used to nondimensionalize the frequency ranges of validity of the infinite shell mobility formulae, where

$$\Omega = \frac{f}{f_{ring}} \tag{11}$$

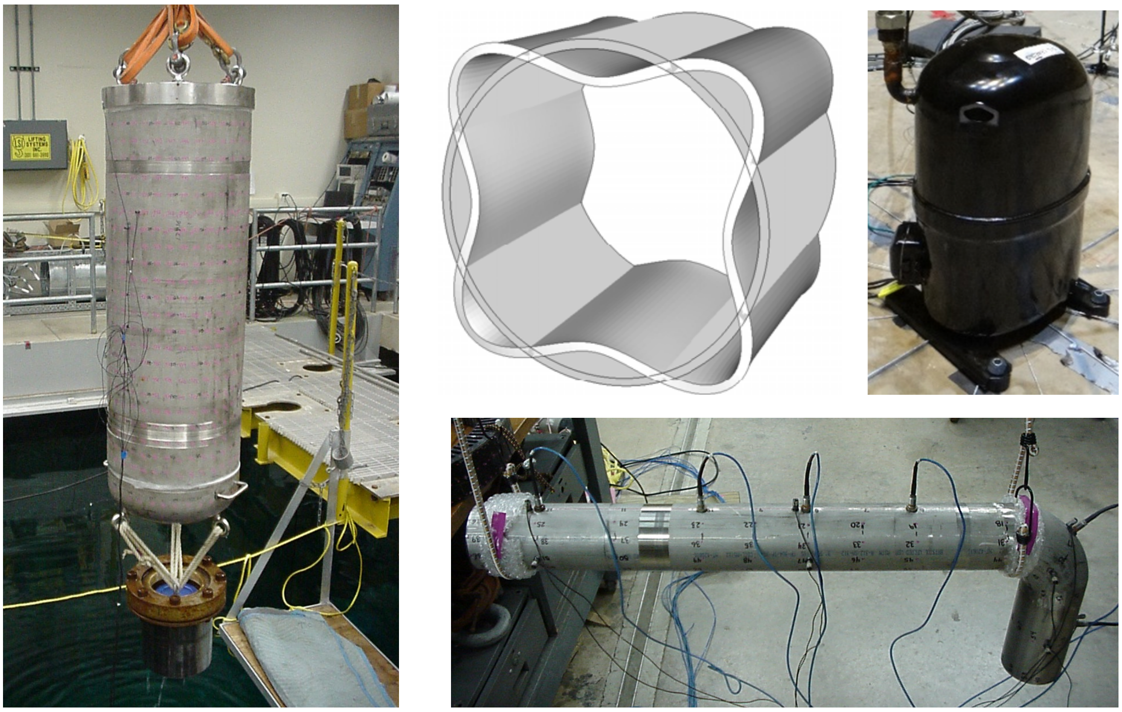

Figure 15: Examples of cylindrical shell structures.

Here are the cylindrical shell infinite mobility equations:

$$
\begin{aligned}
&= \frac{1}{4\pi a \rho_s h \sqrt{\frac{\Omega c_l^2}{\sqrt{2}}}} && \Omega < 0.77\frac{h}{a} && (12)\\
(v/F)_{inf} &= \frac{0.66}{2.3 c_l \rho_s h^2}\sqrt{\Omega} && 0.77\frac{h}{a} < \Omega < 0.6 && (13)\\
&= \frac{1}{8\sqrt{D\rho_s h}} && \Omega > 0.6 && (14)
\end{aligned}
$$

The first equation is for low frequencies, and represents beam-like motion of the shell, which is dominant at frequencies $\Omega < 0.77(h/a)$. This frequency corresponds (roughly) to the cut on of *lobar* modes, where the shell walls deform around the circumference (for more details see [1]). Above Ω of about 0.6, the wavelengths become small with respect to the shell curvature, and the waves behave as they do in an infinite flat plate. The frequency ranges of applicability for these equations is approximate only - it's fine to adjust them slightly based on observed mobility data.

Let's look at some examples, starting with the pipe mobilities shown in Figure 16. This shell has an L/a of 23, an h/a of 0.123, and a ring frequency of 18.8 kHz. The mobility up to about 2 kHz is beamlike, with lobar modes appearing above that frequency. In this example, only the infinite beam and shell mobilities are shown, since Ω of 0.6 is much higher than the upper frequency range of these plots. Mobilities are shown in air and in water, with the water loading shifting the modes lower in frequency. Therefore, to use the equations above, the mass loading of the water should be approximated and added to the structural mass density, and the frequency ranges of applicability should be adjusted slightly lower. Note that the beam-like mobility decreases with the square root of increasing frequency, consistent with the observations made about Equations 3 and 4. The shell-like mobility, however, increases with the square root of increasing frequency.

The mobilities for the large cylindrical shell (L/a of 5.3, h/a of 0.042, and a ring frequency of 3.6 kHz) are shown in Figure 17. Here, since h/a is much smaller, shell-like motion is dominant even at low frequencies. Close to the ring frequency the mobilities become less frequency dependent, approaching that of an infinite plate. Once again, mobilities in air and in water are shown, with the water loading not only adding mass to shift the resonance frequencies downward, but also adding damping due to sound radiation.

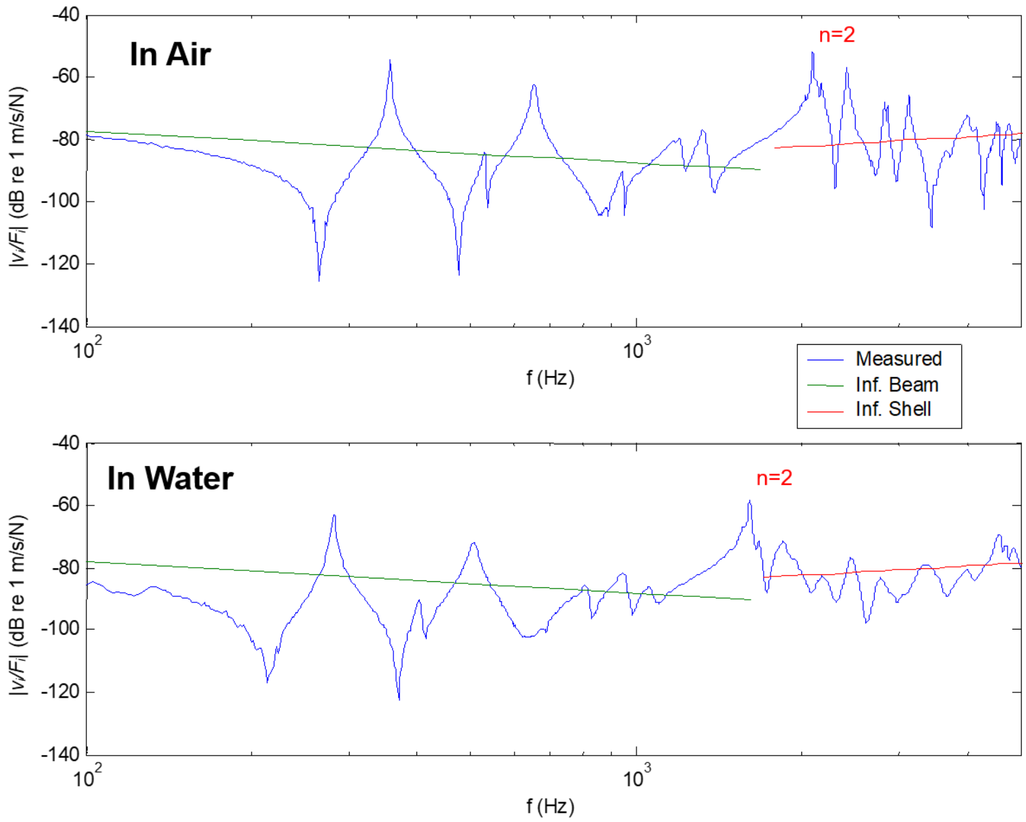

Figure 16: Mobilities of pipe with elbow in air (top) and submerged in water (bottom) compared to infinite beam and shell mobilities.

These examples confirm that the infinite shell mobilities not only represent the geometric mean of the finite shell mobilities (Amazing Uses 1 and 2), but::

Amazing Use 6: Infinite cylindrical shell mobility equations identify the beam-like, shell-like, and plate-like frequency ranges of finite shell mobility response

So, given only the shell geometry and its ring frequency, you can approximate the regions of beam, shell, and plate like behavior! Let's prove this one more time with the compressor shown in Figure 15. Figure 18 shows several drive point mobilities measured at various locations on the compressor housing. In this example, all three frequency ranges of infinite shell mobility behavior are evident. The compressor shell is actually elliptical, causing the mobility dips near Ω of 0.3 to shift in frequency depending on location.

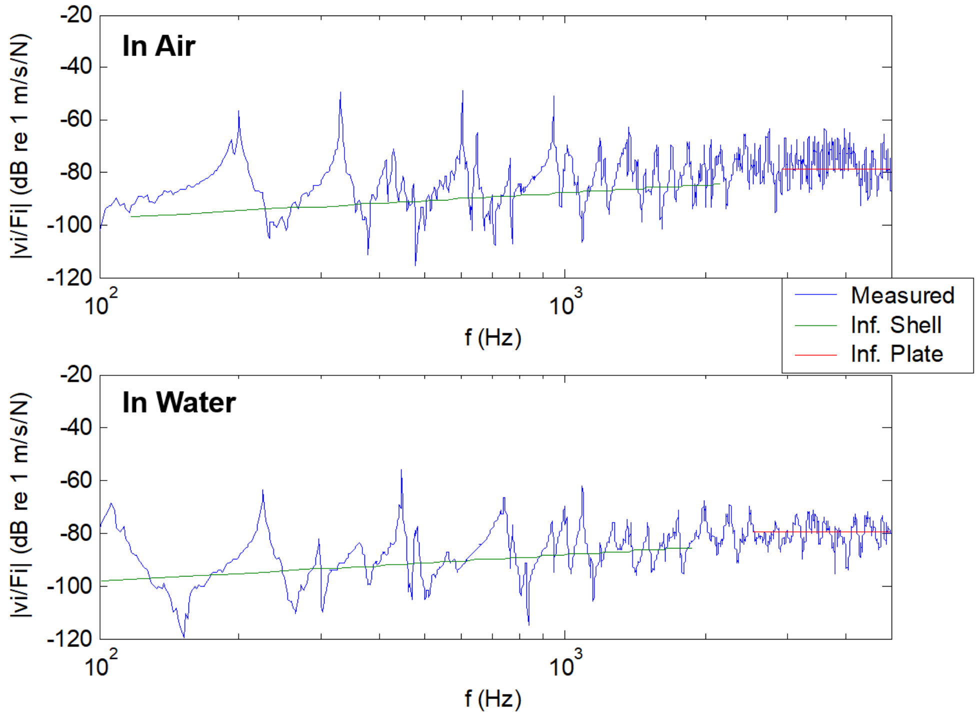

Figure 17: Mobilities of large cylindrical shell in air (top) and submerged in water (bottom) compared to infinite shell mobilities.

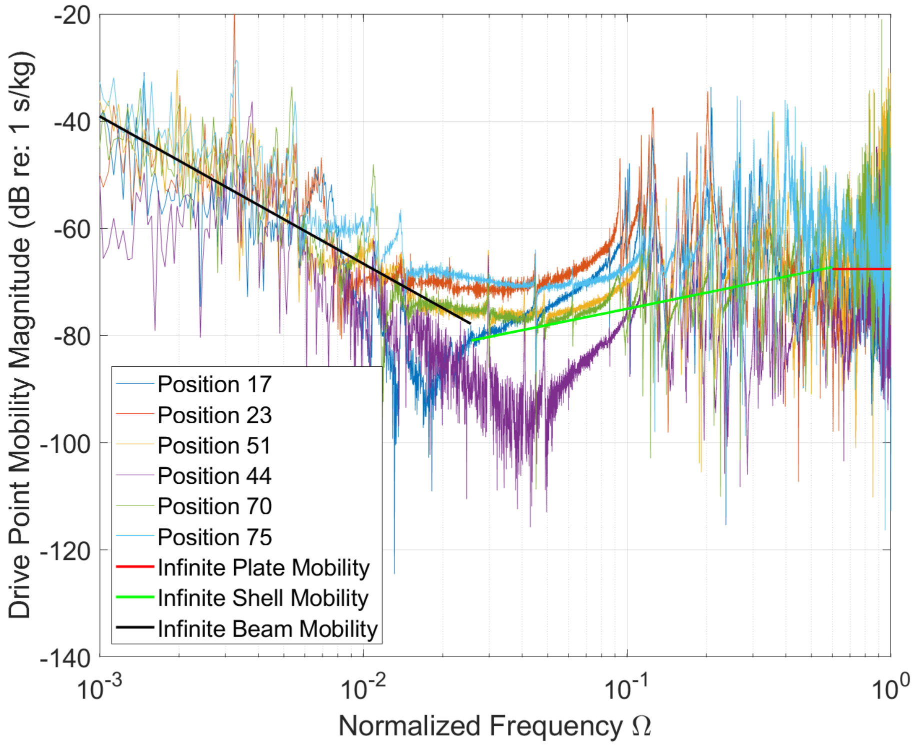

Figure 18: Mobilities of reciprocating compressor compared to infinite shell mobilities.

6. SUMMARY

I hope after reading this tutorial you are now a believer in the Amazing Uses of infinite structure theory. These formulae do far more than approximate the mean mobilities of beams, panels, and shells, also allowing you to:

– check the accuracy of measured mobilities,

– scale mobilities for geometrically similar structures manufactured of different materials,

– estimate mobilities at low and high frequencies of sandwich panels and identify the frequency where shear waves become dominant, and

– estimate mobilities of cylindrical shells over all frequencies, and identify the frequency ranges of beam-like, shell-like, and flat plate-like response.

The principles behind infinite structure theory have been extended significantly in the numerical analysis methodology *Statistical Energy Analysis (SEA)*. There are many references on SEA, and you can find an overview by Shorter and Cotoni in [8]. The concept of *modal density* is related directly to the mobilities described here, and additional formulations allow computing the mean response of interconnected structures and acoustic spaces. Further extensions allow estimates of the spatial variance of the response as well.

7. ACKNOWLEGEMENTS

A reminder that I developed none of the infinite structure mobility formulae in this paper - all credit goes to Drs. Cremer, Heckl, Ungar, [6] and Skudrzyk [7]. Also, all of the measured mobilities and most of the plots were made by dedicated graduate students over the years, including Ben Doty (large shell [9] and elbowed pipe [10]), Andrew Munro (ribbed panels [11]), Micah Shepherd (sandwich panel [4]), and John Cunsolo (compressor [12]). Finally, I will always be indebted to Dr. Yun-Fan Hwang for first introducing me to infinite structure theory many years ago.

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