MADRID inter.noise 2019 June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

Sound zone reproduction using loudspeaker array

Yang Jun[1] Wu Ming[2] Yang Feiran[3] and Han Zerui[4]
Key Laboratory of Noise and Vibration Research, Institute of Acoustics, Chinese Academy of Sciences
Beijing 100190, China

ABSTRACT

Reproduction of a desired sound field over the target region is a hot topic in the research area of the spatial audio. For multi-zone sound field reproduction (SFR), I) an algorithm integrated a Least-Square (LS) criteria with Acoustic contrast control (ACC) constraint is proposed, which tunes the balance between the acoustic contrast and the spatial average error, II) two time-domain ACC methods based on response variation and differential constraints are introduced, respectively, which can avoid the causality problem and maintain a flat frequency response in the "bright" zone. Moreover, the issues around ensuring robust performance in SFR systems are investigated. A framework for robust SFR technique is developed, which allows a physical perspective on the regularization required for a system, increases robustness of the SFR systems against perturbations, and simplifies the SFR system design. For single-zone SFR, a time-domain SFR approach using the group Lasso is presented, which achieves an accurate SFR performance over the target region using a small number of activated loudspeakers.

Keywords: Sound field reproduction, Loudspeaker array
I-INCE Classification of Subject Number: 74

1. INTRODUCTION

The Ambisonics [1], wave field synthesis [2] (WFS), and LS [3] techniques in SFR focus on the reproduction performance of large-scale sound field regions, that is, the similarity between the reproduced sound field and the target sound field, which is generally defined as the normalized mean square error (MSE). These technologies

[1]jyang@mail.ioa.ac.cn
*Author to whom correspondence should be addressed.
[2]mingwu@mail.ioa.ac.cn
[3]feiran@mail.ioa.ac.cn
[4]hanzerui@mail.ioa.ac.cn
**Also at: School of Electronic, Electrical and Communication Engineering, University of Chinese Academy of Sciences, Beijing 100049, China.

is mainly used in large venues and aim to provide better listening experience to more listeners, so larger listening area is preferred and the performance of the sound focusing is omitted. In contrast, the ACC method concentrates on the acoustic contrast performance, and reproduce the sound field only in a local area by maximizing the ratio of the average acoustic energy [4]. The optimization cost function of the ACC method indicates that it only focuses on the contrast performance rather than the MSE, resulting in poor sound quality. However, in practice, spatial average error should be considered together with acoustic contrast in the design of a local sound field that produces high-quality sound around the user. Wu and Abhayapala proposed a multi-zone reproduction technique [5] for reproducing sound in bright zone while limiting the acoustic potential energy in dark zone, taking into account two performance metrics. Chang and Jacobson proposed the acoustic contrast control pressure matching (ACC-PM) method and introduced weighting factor parameters to adjust the trade-off between the two performance indicators [6]. However, the relationship between weighting factors and acoustic contrast is still unclear. This paper proposes a method that combines LS-based SFR with ACC constraints and introduces constraint parameters that represent minimum acoustic contrast [7]. The spatial average error is minimized only when the acoustic contrast is greater than the value of the constraint parameter.

Traditional ACC method is typically designed on a set of discrete frequencies and transformed to the time domain using an inverse discrete Fourier transform. The resulting design cannot avoid causality problems and it is not possible to obtain satisfactory acoustic contrast over a broad frequency range, especially when the filter length is short. To solve these problems, Elliott and Cheer first designed ACC in the time domain and proposed the broadband acoustic contrast control (BACC) method [8]. Although the BACC method can solve broadband contrast problems and causal problems, the frequency response of bright zone cannot be controlled; this may cause frequency response distortion. In this paper, the response variation (RV) and response differential (RD) terms are introduced in the BACC-RV method and BACC-RD method to overcome this problem [9]. The improved methods have good contrast performance over a wide frequency range and reproduce a flat frequency response in the bright zone.

SFR systems are susceptible to the interference of the acoustic transfer functions (ATFs) between the loudspeaker array and the control microphones, which may be due to the sensitivity of each source, the complexity of the spatial response or the mismatch of the source location. In order to prevent performance degradation under real conditions, the robustness should be carefully considered. In this paper, a robust acoustic modeling-based SFR framework is developed [10–12]. The acoustic modeling describes the ideal radiation from the loudspeakers to the control microphones, and uses the estimated error model to represent the amplitude and phase variations in the ATFs. In addition, a simplified model-based estimation of regularization parameters is proposed, which makes robust ACC more practical.

SFR systems using LS methods usually activate all candidate loudspeakers to minimize MSE between desired and reproduced sound fields, which may result in blurred spatial sound images [13]. Although the least-absolute shrinkage and selection operator (Lasso) can be used to limit the number of active loudspeakers [14], this algorithm is designed in the frequency domain. In this paper, we present a time-domain SFR method based on the framework of the group Lasso [15] (GL) which is an extension of the Lasso to select grouped variables in linear regression models [16]. The proposed method is first described as an optimization problem with mixed-norm constraint and then iteratively solved using the block coordinate descent [17] (BCD) algorithm. In addition, the GL method can also be used to optimize the loudspeaker arrangement of the SFR system.

2. MULTI-ZONE SOUND FIELD REPRODUCTION

A multi-zone sound field reproduction(SFR) is proposed to generate desired sound in a bright zone while maintaining silence in a dark zone as shown in Fig. 1. One common used multi-zone sound field reproduction method is acoustic contrast control (ACC). In this sections, several methods is proposed to improve the performance of the acoustic contrast control.

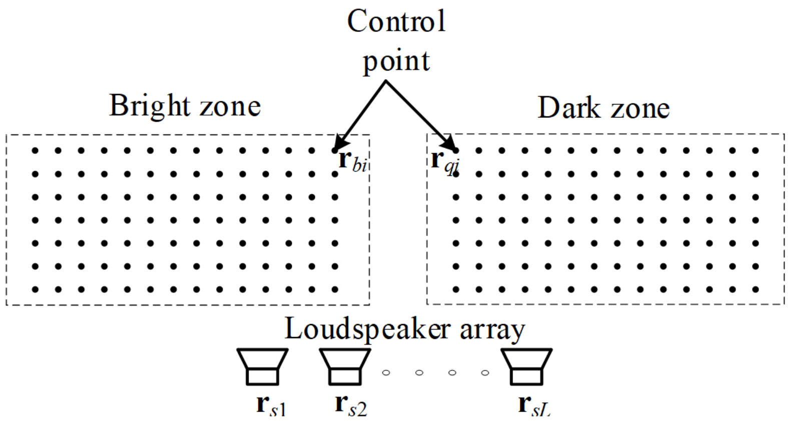


Figure 1: The diagram of the multi-zone sound field reproduction system

2.2.1. Least-squares approach with acoustic contrast control constraint

Acoustic contrast and the spatial average error are usually used to evaluate the performance of the multi-zone sound field reproduction method. In [7], we proposed the SFR-ACC algorithm which minimized the spatial average error when the acoustic contrast is larger than the value of constrained parameter

$$\min_{\mathbf{w}} \|\mathbf{G}_{\mathrm{B}}\mathbf{w}\text{-}\mathbf{d}\|_2^2 \tag{1}$$

$$\text{s.t.} \quad 10\log_{10}C \geq 10\log_{10}C_{\mathrm{con}} \tag{2}$$

where

$$\mathbf{G}_{\mathrm{B}}(\omega) = [\mathbf{g}(\mathbf{r}_{b1}|\mathbf{r}_{\mathrm{s}},\omega),\mathbf{g}(\mathbf{r}_{b2}|\mathbf{r}_{\mathrm{s}},\omega)\cdots\mathbf{g}(\mathbf{r}_{bM}|\mathbf{r}_{\mathrm{s}},\omega)]^T \tag{3}$$

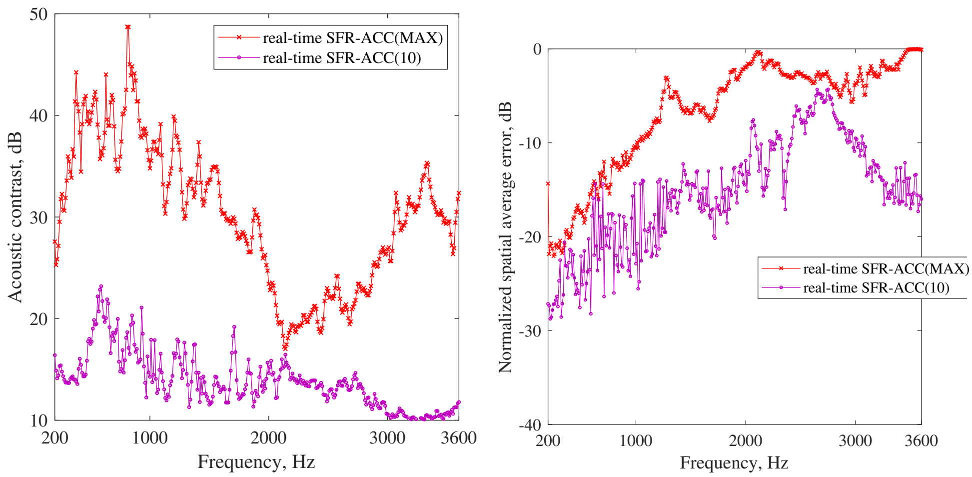
$$\mathbf{g}(\mathbf{r}_{bi}|\mathbf{r}_s,\omega) = [g(\mathbf{r}_{bi}|\mathbf{r}_{s1},\omega), g(\mathbf{r}_{bi}|\mathbf{r}_{s2},\omega)\cdots g(\mathbf{r}_{bi}|\mathbf{r}_{sL},\omega)]^T \tag{4}$$

ω is the angular frequency; $g(\mathbf{r}_{bi}|\mathbf{r}_{sj},\omega)$ denotes the transfer function between the jth loudspeaker element and the ith control point; $\mathbf{w}(\omega) = [w_1(\omega),w_2(\omega)\cdots w_L(\omega)]^T$ is the weight vector of the loudspeaker array, $\|\cdot\|_2$ represents the ℓ_2 norm, and $\mathbf{d}(\omega) = [p_d(\mathbf{r}_{b1},\omega), p_d(\mathbf{r}_{b2},\omega)\cdots p_d(\mathbf{r}_{bM},\omega)]^T$ is the desired sound field,

$$C = \frac{\mathbf{w}^H\mathbf{R}_{\mathrm{B}}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{\mathrm{D}}\mathbf{w}} \tag{5}$$

denotes the acoustic contrast, and C_{con} is the constrained parameter which denotes the minimum allowable acoustic contrast in the sound reproduction system. $\mathbf{R}_B = \mathbf{G}_b{}^H \mathbf{G}_b$ is the spatial correlation matrix in the bright zone, and $\mathbf{R}_D$ is the spatial correlation matrix in the dark zone. The cost function of SFR-ACC method defined in Equation 1 and Equation 2 can be converted to a semi-definite program problem and solved through using the convex toolbox [18].

To evaluate the performance of the SFR-ACC method, experiments are carried out in the anechoic chamber. The linear loudspeaker array consists of eight moving-coil speaker units spaced 12 cm apart. The bright zone and dark zone are located at the −45° and 45° direction deviated from the loudspeaker array center, respectively. Fig. 2 shows the acoustic contrast and spatial average error when C_{con} is set to different value. For SFR-ACC(MAX) and SFR-ACC(10), C_{con} are set to $10\log_{10}C_{max}$ and 10dB, respectively. It can be seen that the proposed SFR-ACC method allows a trade off between acoustic contrast and spatial average error. For SFR-ACC(MAX), the maximum acoustic contrast can be achieved. By reducing acoustic contrast, SFR-ACC(10) can have a lower spatial average error compared with SFR-ACC(MAX). Since C_{con} is set to 10 dB, the acoustic contrast of SFR-ACC(10) is always larger than the level of 10 dB as expected.



(a) (b)

Figure 2: (a) Acoustic contrast for SFR-ACC(MAX) and SFR-ACC(10) and (b) Spatial average error for SFR-ACC(MAX) and SFR-ACC(10)

2.2.2. Time-domain multi-zone sound field reproduction method

The traditional multi-zone sound field reproduction approach is usually designed in frequency domain. However, when the length of w is short, the performance at frequencies without control is seriously degraded. Time-domain acoustic contrast control (BACC) method is proposed to solve this problem. The structure of BACC is illustrated in Fig. 3, where each of the L loudspeakers is driven by the output of a finite impulse response (FIR) filter $w_l(n)$. All of the filters have the same length M. The contrast problem can be alleviated by the BACC approach. However, the frequency response in the bright zone may be serious distorted. To tackle this problem, we propose two time-domain acoustic contrast control methods denotes as BACC-RV and BACC-RD in [9].

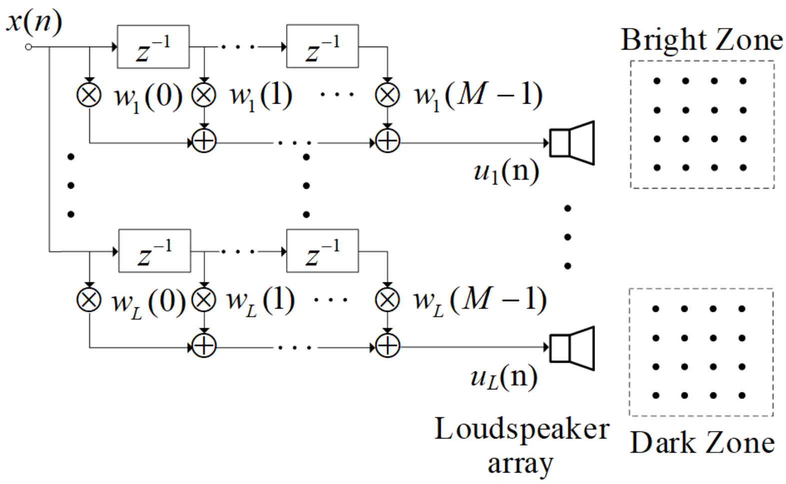


Figure 3: The structure of BACC algorithm

The BACC-RV method aims to solve the problem

$$\max_{\mathbf{w}} \frac{\mathbf{w}^T\mathbf{R}_{\mathrm{B}}\mathbf{w}}{\beta\mathbf{w}^T\mathbf{R}_{\mathrm{D}}\mathbf{w} + (1-\beta)\mathrm{RV} + \delta\mathbf{w}^T\mathbf{w}}, \tag{6}$$

where

$$\mathbf{w} = [w_1(0), \cdots, w_1(M-1), \cdots, w_L(0), \cdots, w_L(M-1)]^T \tag{7}$$

is the $ML \times 1$ coefficient vector, $\mathbf{R}_{\mathrm{B}} = \sum_{k=1}^{K}\sum_{n=0}^{M+I-2} \mathbf{r}_{\mathrm{B}k}(n)\mathbf{r}_{\mathrm{B}k}^T(n)/K$ is the normalized correlation matrix in the bright zone,

$$\mathbf{r}_{\mathrm{B}k}(n) = [h_{\mathrm{B}1k}(n), \cdots, h_{\mathrm{B}1k}(n-M+1), \cdots, h_{\mathrm{B}Lk}(n), \cdots, h_{\mathrm{B}Lk}(n-M+1)]^T \tag{8}$$

is the filtered signal vector, $h_{\mathrm{B}lk}(n)$ denotes the impulse response between the lth loudspeaker and kth control point, $\mathbf{R}_{\mathrm{D}}$ the normalized correlation matrix in the dark zone β is a weight factor whose value is between 0 and 1. The RV term is introduced in Equation 6 to measure the response variation over the frequency range of interest in the bright zone

$$\mathrm{RV} = \frac{1}{JK}\sum_{k=1}^{K}\sum_{j=1}^{J}\left|p_{\mathrm{B}k}(f_j) - p_{\mathrm{B}k}(f_{\mathrm{ref}})\right|^2 \tag{9}$$

where $p_{\mathrm{B}k}(f)$ is the frequency response at the kth control point in the bright zone and f_{ref} is the reference frequency.

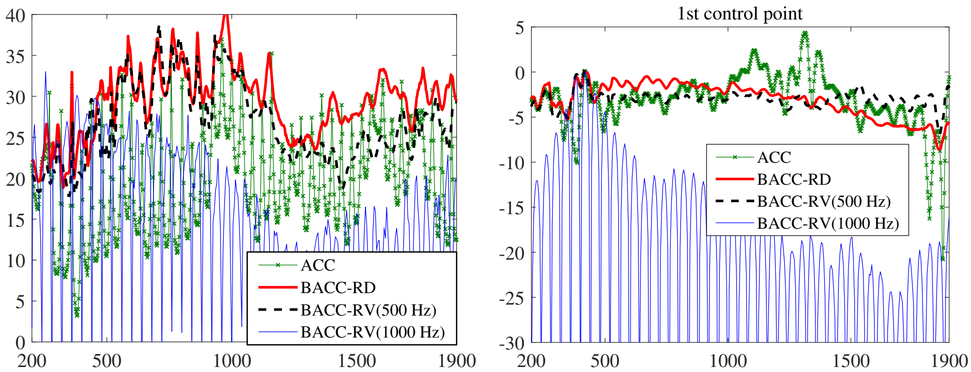
It can be seen from Equation 6 that in the BACC-RV approach, the reference frequency need to be selected carefully since the performance of acoustic contrast is sensitive to the reference frequency. To address this problem, the BACC-RD approach is proposed

$$\max_{\mathbf{w}} \frac{\mathbf{w}^T\mathbf{R}_{\mathrm{B}}\mathbf{w}}{\beta\mathbf{w}^T\mathbf{R}_{\mathrm{D}}\mathbf{w} + (1-\beta)\mathrm{RD} + \delta\mathbf{w}^T\mathbf{w}} \tag{10}$$

where $\mathrm{RD} = \frac{1}{(J-1)K}\sum_{k=1}^{K}\sum_{j=1}^{J-1}\left|p_{\mathrm{B}k}(f_{j+1}) - p_{\mathrm{B}k}(f_j)\right|^2$ is defined as the mean square of the first-order differential of the frequency response in the bright zone.

Experiments are carried out in the anechoic chamber to validate the proposed methods. The bright and dark zones are located at the −45° and 45° directions deviated from the center, respectively. The loudspeaker array consists of eight loudspeaker units with a spacing d of 10 cm. Figure 4(a) illustrates the resulting acoustic contrast against the frequency for the ACC, BACC-RD, BACC-RV(500 Hz), and BACC-RV(1000 Hz) methods, where 500 Hz and 1000 Hz stand for the reference frequency. It can be seen that conventional ACC approach can only achieve good acoustic contrast at a discrete set of control frequencies. This is because only the information at the control frequencies is used and contrast at other frequencies cannot be controlled. In contrast, the BACC-RV(500 Hz) and BACC-RD methods are directly designed in the time-domain and use the information at all frequencies points. Therefore, both of them are able to get satisfactory contrast over the whole frequency range. It can be seen that, compared with BACC-RV(500 Hz), a better contrast at higher frequencies can be achieved by BACC-RD.

The frequency response in the bright zone are plotted in Fig. 4(b). All of these methods except for BACC-RV(1000 Hz) can achieve flat frequency response at the center of the bright zone. However, it can be observed that the performance of ACC is degraded at the edge of zone, while the BACC-RV(500 Hz) and BACC-RD methods can still yield a well flat response at the edge of zone.



(a) (b)

Figure 4: The experimental performance of the ACC, BACC-RV and BACC-RD methods, where the reference frequency in the BACC-RV method is 500 Hz and 1000 Hz, respectively, (a) acoustic contrast, and (b) frequency response in the bright zone

2.2.3. Robust multi-zone sound field reproduction method

In order to prevent performance degradation under realistic conditions in methods directly based on transfer functions, robustness should be carefully considered in the algorithm design. In [10–12], we model the acoustic transfer functions from the sources to the control zones as

$$\tilde{\mathbf{G}} = \mathbf{G} + \Delta\mathbf{G} \tag{11}$$

where G is the determined part and ΔG is a set of potential error which describes variances in the transfer functions.

Worst-case optimization (WCO) aims to find and optimize the situation giving the worst performance among all the possible probabilities

$$\max_{\mathrm{w}} \min_{\Delta\mathbf{R}_{\mathrm{D}}} \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{B}} \mathbf{w}}{\mathbf{w}^H \tilde{\mathbf{R}}_{\mathrm{D}} \mathbf{w}} \quad \text{s.t. } \|\Delta\mathbf{R}_{\mathrm{D}}\|_F \leq \delta_Q \tag{12}$$

where $\tilde{\mathbf{R}}_{\mathrm{D}} = \mathbf{R}_{\mathrm{D}} + \Delta\mathbf{R}_{\mathrm{D}}$, $\Delta\mathbf{R}_{\mathrm{D}}$ represents the perturbations in the spatial correlation matrix, δ_Q is the bound of $\Delta\mathbf{R}_{\mathrm{D}}$. The solution of Equation 12 is

$$\mathbf{w}_{\mathrm{wc}}=\mathrm{Eig1}\left\{(\mathbf{R}_{\mathrm{D}} + \delta_{\mathrm{D}}\mathbf{I})^{-1}\mathbf{R}_{\mathrm{B}}\right\} \tag{13}$$

where Eig1$\{\cdot\}$ denotes the operator that yields the principal eigenvector corresponding to the maximum eigenvalue and $\mathbf{I}$ is a unit matrix.

Probability-model optimization (PMO) aims to improve the average performance according to the distribution. For multiplicative transfer function error, the solution is

$$\mathbf{w}_{\mathrm{ME}}=\mathrm{Eig1}\left\{(\mathbf{R}_{\mathrm{D}} \otimes \mathbf{E}_{\mathrm{D}})^{-1}(\mathbf{R}_{\mathrm{B}} \otimes \mathbf{E}_{\mathrm{B}})\right\} \tag{14}$$

where $\otimes$ is pointwise multiplication. $\mathbf{E}_{\mathrm{B}}$ and $\mathbf{E}_{\mathbf{D}}$ are the statistical features of the transfer function errors. For additive transfer function errors with uniform distribution, the solution is

$$\mathbf{w}_{\mathrm{ME}}=\mathrm{Eig1}\left\{\left(\mathbf{R}_{\mathrm{D}} + \frac{Ma_{\max}^2}{3}\mathbf{I}\right)^{-1}\left(\mathbf{R}_{\mathrm{B}} + \frac{Ma_{\max}^2}{3}\mathbf{I}\right)\right\} \tag{15}$$

where $a_{\max}$ is the maximum amplitude of the additive transfer function error.

The simulation is carried out to compare the robust control approaches. An arc-shaped array with 11 loudspeakers is adopted. The loudspeakers are uniformly arranged with a distribution angle of 6°. The control points for the listening and quiet zones are defined on dual circles, composed of 24 microphones in each ring, with radii of 0.083 m and 0.104 m. the transfer function is assume to be multiplicative form with Gaussian distribution between -3 dB and +3 dB in amplitude and uniform distribution between $-10°$ and $+10°$ in phase.

Fig. 5 shows the mean acoustic contrast (AC) and array effort (AE) performance for ACC algorithm with no regularization (NR), with maximum singular value related regularization (SV), with array effort limited to 0 dB (EL0), with array effort minimized (ELM) and AEQ. The parameters estimated for WCQ and MEQ gave very similar results to AEQ (except that MEQ was slightly better at low frequencies) and are therefore omitted from the figure for clarity. It can be seen all robust approaches improve performance over NR. Among them, AEQ is best, both in terms of AC and AE at the frequency band ranged from 100 to 8000 Hz.

3. SINGLE ZONE SOUND FIELD REPRODUCTION

In many applications, it is desired to reproduce a sound field over a predefined spatial region using the loudspeaker array. Both data-based and model-based solutions have been proposed to address this problem in the past years. In the model-based system, it is the desired field of each sound model can be generated by a small number of loudspeakers.

We present the sound field reproduction system in Fig. 6. The input signal $s_0(m)$ is filtered using a FIR filter $w_q(l)$ before it is sent to the q-th loudspeaker with $q \in \{1, 2, \ldots, Q\}$. The impulse response between the q-th loudspeaker and the n-th control point is denoted by $h_{qn}(j)$ with $n \in \{1, 2, \ldots, N\}$ and $j \in \{0, 1, \ldots, J-1\}$. Assuming that the input signal as a unit impulse $s_0(m) = \delta(m)$, the reproduced sound pressure at the n-th control point is

$$p_n^r(m) = \mathbf{h}_n^T(m)\mathbf{w} \tag{16}$$

where $\mathbf{w} = [w_1(0), \cdots, w_1(L-1), \cdots, w_Q(0), \cdots, w_Q(L-1)]^T$ is the unknown coefficient vector with a length LQ, and $\mathbf{h}_n(m) = [h_{1n}(m), \cdots, h_{1n}(m-L+1), \cdots, h_{Qn}(m), \cdots, h_{Qn}(m-$

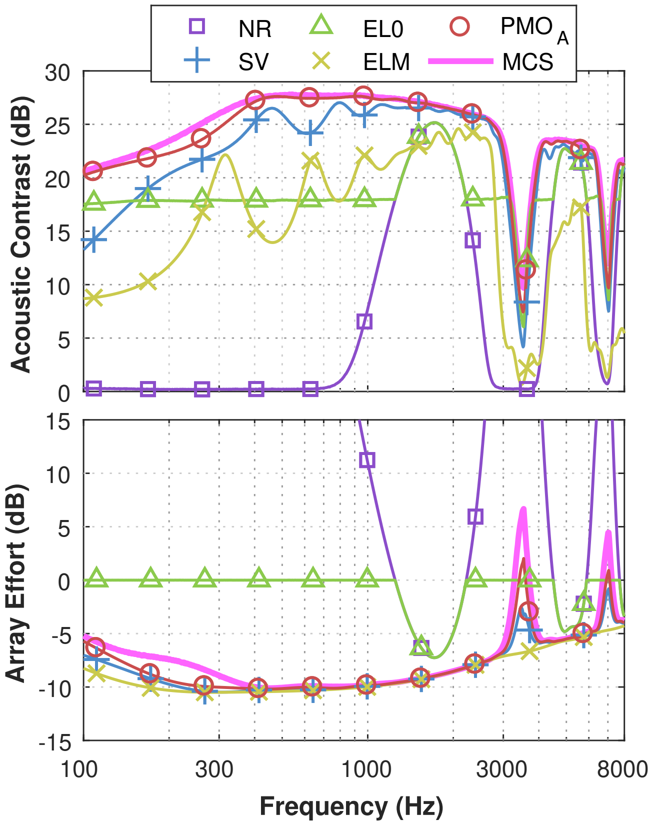


Figure 5: The mean acoustic contrast (AC) and array effort (AE) performance for ACC algorithm with no regularization (NR), with maximum singular value related regularization (SV), with array effort limited to 0 dB (EL0), with array effort minimized (ELM) and AEQ.

$L + 1)]^T$. The vector which is constructed by the reproduced sound pressures at N control points reads

$$\mathbf{p}_r(m) = \mathbf{H}(m)\mathbf{w} \tag{17}$$

where $\mathbf{p}_r(m) = [p_1^r(m), \cdots, p_N^r(m)]^T$ and

$$\mathbf{H}_m = \begin{bmatrix} \mathbf{h}_1^T(m) \\ \vdots \\ \mathbf{h}_N^T(m) \end{bmatrix}. \tag{18}$$

Define a vector which is constructed by the desired sound pressures at all the control points as $\mathbf{p}_d(m) = [p_1^d(m), \cdots, p_N^d(m)]^T$. We then define the following cost function

$$E = \sum_{m=0}^{M-1} ||\mathbf{H}_m\mathbf{w} - \mathbf{p}_d(m)||^2 \tag{19}$$

where $M = L + J - 1$. The least-squares (LS) method can be used to solve this problem. Usually, a constraint is added to the cost function in order to limit the output power of the loudspeaker array

$$\mathbf{w}_{LS} = \arg\min_{\mathbf{w}} \left(\frac{1}{2}E + \lambda||\mathbf{w}||^2 \right). \tag{20}$$

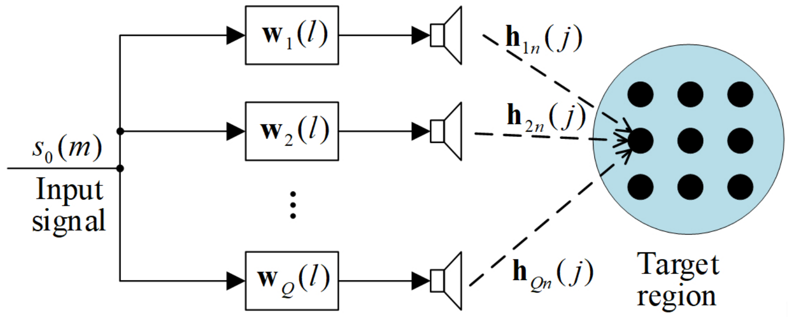


Figure 6: Block diagram of signal processing for sound field reproduction.

However, the LS method usually adopts all the loudspeakers to minimize the cost function. Several methods have been proposed to limit the number of active loudspeakers in the frequency domain [14, 19]. In [16], we proposed a time-domain method under the framework of the group Lasso. We could use the optimize the following optimization problem

$$\mathbf{w}_0 = \arg\min_{\mathbf{w}} \left(\frac{1}{2} E + \gamma ||\mathbf{w}||_0 \right), \tag{21}$$

where γ is the penalty parameters. A more convenient model is to use the group Lasso, i.e.,

$$\mathbf{w}_{\mathrm{GL}} = \arg\min_{\mathbf{w}} \left(\frac{1}{2} E + \gamma ||\mathbf{w}||_1 \right). \tag{22}$$

This problem can be solved by the BCD method. The advantage of this method lies in two aspects. First, the number of active loudspeakers is greatly reduced. Second, the group Lasso method can optimizes all the control frequencies simultaneously and also avoid the causality problem.

4. CONCLUSIONS

This paper introduces the latest research progress of the Key Laboratory of Noise and Vibration Research in the direction of sound field reproduction in recent years, including the following work [7, 9–11, 16]:

An algorithm integrated a Least-Squares (LS) criteria with ACC constraint is proposed. Experimental results verified that the spatial average error is minimized only when the minimum acoustic contrast is guaranteed in the proposed method. Furthermore, the proposed approach can improve the flatness of response in the bright zone by sacrificing the level of acoustic contrast.

Two time-domain ACC design based on response variation and differential constraints has been proposed, respectively. Compared with the frequency domain ACC, the proposed method can avoid the causality problem and provides excellent acoustic contrast over the continuous frequency and maintain a flat frequency response.

A framework is proposed for robust sound zone reproduction design, which allows a physical perspective on the regularization required for a system. Robust ACC is formulated and implemented with a simple error model, adopting robust optimization strategies (PMO and WCO).

A time-domain SRF method based on the Group LASSO (GL) is proposed, which optimizes the positions and the number of activated loudspeakers in the time domain.

The proposed method can achieve an accurate broadband SFR by using a small number of loudspeakers, and its corresponding two-stage algorithm GL-LS can provide better SFR than the one-stage methods

5. ACKNOWLEDGMENTS

This research was supported by the National Key Research and Development Program 307 of China (Grant No. 2016YFB1200503).

6. REFERENCES

[1] Darren B Ward and Thushara D Abhayapala. Reproduction of a plane-wave sound field using an array of loudspeakers. *IEEE Transactions on speech and audio processing*, 9(6):697–707, 2001.

[2] Augustinus J Berkhout, Diemer de Vries, and Peter Vogel. Acoustic control by wave field synthesis. *The Journal of the Acoustical Society of America*, 93(5):2764–2778, 1993.

[3] Mark Poletti. An investigation of 2-d multizone surround sound systems. In *Audio Engineering Society Convention 125*. Audio Engineering Society, 2008.

[4] Joung-Woo Choi and Yang-Hann Kim. Generation of an acoustically bright zone with an illuminated region using multiple sources. *The Journal of the Acoustical Society of America*, 111(4):1695–1700, 2002.

[5] Yan Jennifer Wu and Thushara D Abhayapala. Spatial multizone soundfield reproduction: Theory and design. *IEEE Transactions on audio, speech, and language processing*, 19(6):1711–1720, 2011.

[6] Ji-Ho Chang and Finn Jacobsen. Sound field control with a circular double-layer array of loudspeakers. *The Journal of the Acoustical Society of America*, 131(6):4518–4525, 2012.

[7] Yefeng Cai, Ming Wu, and Jun Yang. Sound reproduction in personal audio systems using the least-squares approach with acoustic contrast control constraint. *The Journal of the Acoustical Society of America*, 135(2):734–741, 2014.

[8] Stephen J Elliott and Jordan Cheer. Regularisation and robustness of personal audio systems. 2011.

[9] Yefeng Cai, Ming Wu, Li Liu, and Jun Yang. Time-domain acoustic contrast control design with response differential constraint in personal audio systems. *The Journal of the Acoustical Society of America*, 135(6):EL252–EL257, 2014.

[10] Qiaoxi Zhu, Philip Coleman, Ming Wu, and Jun Yang. Robust personal audio reproduction based on acoustic transfer function modelling. In *Audio Engineering Society Conference: 2016 AES International Conference on Sound Field Control*. Audio Engineering Society, 2016.

[11] Qiaoxi Zhu, Philip Coleman, Ming Wu, and Jun Yang. Robust acoustic contrast control with reduced in-situ measurement by acoustic modeling. *Journal of the Audio Engineering Society*, 65(6):460–473, 2017.

[12] Qiaoxi Zhu, Philip Coleman, Xiaojun Qiu, Ming Wu, Jun Yang, and Ian Burnett. Robust personal audio geometry optimization in the svd-based modal domain. *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 27(3):610–620, 2019.

[13] P-A Gauthier, P Lecomte, and A Berry. Source sparsity control of sound field reproduction using the elastic-net and the lasso minimizers. *The Journal of the Acoustical Society of America*, 141(4):2315–2326, 2017.

[14] Georgios N Lilis, Daniele Angelosante, and Georgios B Giannakis. Sound field reproduction using the lasso. *IEEE Transactions on Audio, Speech, and Language Processing*, 18(8):1902–1912, 2010.

[15] Noah Simon, Jerome Friedman, Trevor Hastie, and Robert Tibshirani. A sparse-group lasso. *Journal of Computational and Graphical Statistics*, 22(2):231–245, 2013.

[16] Qipeng Feng, Feiran Yang, and Jun Yang. Time-domain sound field reproduction using the group lasso. *The Journal of the Acoustical Society of America*, 143(2):EL55–EL60, 2018.

[17] Paul Tseng. Convergence of a block coordinate descent method for nondifferentiable minimization. *Journal of optimization theory and applications*, 109(3):475–494, 2001.

[18] Michael Grant and Stephen Boyd. CVX: Matlab software for disciplined convex programming, version 2.1. http://cvxr.com/cvx, March 2014.

[19] Nasim Radmanesh and Ian S Burnett. Generation of isolated wideband sound fields using a combined two-stage lasso-ls algorithm. *IEEE Transactions on Audio, Speech, and Language Processing*, 21(2):378–387, 2013.